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Submarine Physics

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THE submarine is peculiarly an American type of vessel. Though accounts exist of earlier attempts to propel a boat under water, it appears that the first successful venture of this type was that by David Bushnell in America in 1775. His boat was called the *Turtle*, and the shape of its wooden hull was not unlike the shell of its namesake. Everything was operated by hand. Valves admitted water for submergence, and the water was pumped out again for surfacing. Manually operated screws through the top and stern provided vertical and horizontal maneuverability of a sort, and a detachable 200-lb weight was provided for the ever-imminent emergency. There was enough air in the boat to support the operator for about 30 min submergence. The navigating instruments were a compass, pressure gage and sounding line. During the Revolution the vessel carried armament in the form of a detachable blister containing a 150-lb powder charge intended to be attached to the wooden bottom of a surface ship by a large wood screw keyed to a crank inside the boat. This offensive contrivance did not work out just as planned in the one operation against H.M.S. *Eagle*, for apparently the screw failed to take hold in the planking. But when the charge exploded later at some distance from the *Turtle*, the latter was not sunk either, so the honors were even.

Technical developments during the next hundred years led in 1875 to the boat produced by John P. Holland. It was primarily for surface operation, being driven by a petroleum engine, but the small crew could endure short periods of submergence. Holland appreciated certain of the fundamental principles of submarine design, such as the necessity of maintaining approximately neutral buoyancy and of preventing the center of gravity from shifting by compartmenting the water-ballast tanks. Vertical maneuverability was provided by horizontal rudders. The usefulness of Holland's boat was seriously impaired by the basic difficulties of securing information on which to navigate and maneuver when submerged. These and other troubles still dog submarines, but modern submarine design owes much to Holland's early efforts. Submarine tenders in the Navy are still named for the early pioneers, Bushnell and Holland, in honor of their contributions to the development of submarines; and, following the precedent of Bushnell's *Turtle*, submarines themselves are named for aquatic animals of all sorts, from the *Whale* and *Manta* to the *Seahorse* and *Sardine*.

As can be seen from the upswept bow and decking (Fig. 1), submarine design has been derived from that of the surface ship. In fact, today's submarine is really a submersible vessel; most of the time it is on the surface, and it can submerge for only limited periods. Its bow is designed for sea-keeping qualities, and it has a small bridge for navigation. It has twin screws, a Diesel power plant of about 6500 hp for surface

propulsion, and the usual radio, radar and signaling equipment for surface operation. These features are of interest because they show that submarine design cannot be concerned solely with problems peculiar to an undersea vessel; many compromises must be made with surface-ship requirements. When submerged the submarine displaces about 1500 tons, and it then has more in common with the airship or airplane than with surface ships. After "take-off" below the surface it operates in three dimensions, and vertical control and stability are problems of constant concern. Diving and surfacing present many of the same technical features as take-offs and landings, fore and aft diving vanes give the vertical maneuverability of fins and ailerons. Mechanical failures present hazards which distinguish this type of operation from that on the surface of the land or sea.

Submarine physics is concerned chiefly with the differences between the physical properties of the sea and the air, which profoundly affect the difference between the vehicles that are designed to operate in these two mediums. The physical laws are of course the same; but the difference in chemical composition, density, compressibility, and electric conductivity are so large as to impose entirely different requirements on vehicles and equipment. The problems of operation in the sea require a complete, imaginative reorientation for all of us who are accustomed to live and move and communicate in the air. When the diving alarm sounds and the submarine slips beneath the waves, it enters a new and unfamiliar world. A slight increase in pressure on the ears tells us that all openings in the submarine are sealed; and when the surface is left well behind, all sense of motion is lost. No ports are provided for observation as the sunlight does not penetrate far below the surface and the ocean is too murky to make use of searchlights. It is like being in an airplane in a dense fog except that the electric motors make almost no sound. All ordinary contact with the upper world is lost so that navigating and maneuvering is "blind" and carried out by reliance on ingenious devices which have been developed especially for the submarine.

Reluctance to relinquish contact with the medium for which we and all of our mechanical devices are designed, combined with the fact that concealment is afforded by even shallow submergence in the opaque ocean, have led to a tenacious retention of such vestigial organs as the *periscope* and *schnorchel*. The periscope is a triumph of optical design which provides the submariner with a high-quality telescope for viewing the surface and air above him while exposing only a few feet of pipe an inch or so in diameter. The schnorchel, which consists of a double pipe for the Diesel intake and exhaust, projects from the top of the submarine like the neck of a clam. It extends only a few feet above the ocean surface, and a float valve prevents water from entering when it is occasionally submerged in a wave. Although somewhat more

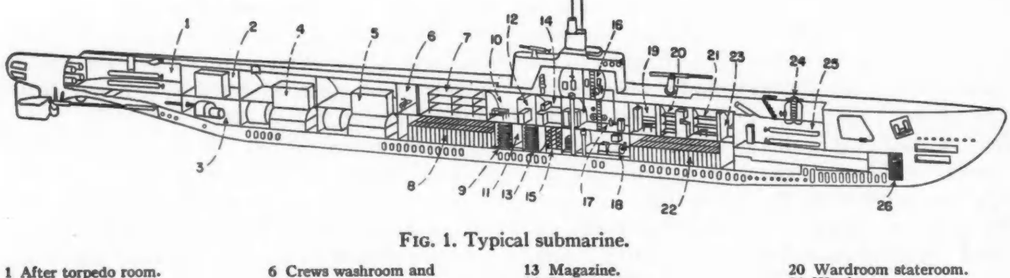


FIG. 1. Typical submarine.

1 After torpedo room.
2 Maneuvering room.
3 Motor room.
4 After machinery compartment.
5 Forward machinery compartment.
6 Crews washroom and shower.
7 Crews quarters.
8 After battery space.
9 Magazine.
10 Mess room.
11 Handling space.
12 Galley.
13 Magazine.
14 Radio room.
15 Storeroom.
16 Conning tower.
17 Control room.
18 Pump room.
19 Chief petty officer's state room.
20 Wardroom stateroom.
21 Ward room.
22 Forward battery space.
23 Pantry.
24 Escape trunk.
25 Forward torpedo room.
26 Chain locker.

conspicuous than the periscope, it permits the submarine to run on its Diesels and thus without battery drain. But these adjuncts reflect lingering efforts to have the submarine remain a surface vessel and are not representative of the features and devices that characterize a true subsurface vessel.

Buoyancy and Vertical Maneuverability

The submarine operates in a medium that is about 800 times more dense than air. Each 10 m of submergence adds a pressure equivalent to 1 atm, so at 200 m below the surface the pressure increment is about 300 lb/in.^{2}, or 20 kg/cm^2. The pressure hull of a submarine is built in the form of an elongated ellipsoid in order that by their curvature the steel plates can withstand the pressure due to this load of water. If the hydrostatic pressure is p, the force per unit length pressing together the two halves separated by the dotted line in Fig. 2 is $2Rp$ at right angles to the figure. This must be supported by the two steel walls of thickness t, so they are under a peripheral pressure pR/t. If the diameter of the submarine is about 6 m and the walls are 3 cm thick, the steel is under a pressure 100 times as great as is the water. Thus a depth of 200 m corresponds to a compression of the steel by a pressure of 2000 kg/cm^2. This leaves only a comfortable factor of safety if 10,000-kg/cm^2 steel is used. All hull openings where the stress becomes larger must be well reinforced, so any unnecessary holes through the pressure hull are regarded with great disfavor. The conning tower, which remains habitable when submerged, is in the form of a cylinder parallel to the hull but shorter and of smaller diameter, and welded securely on top of it. As R is smaller for the conning tower than for the hull, the stresses in the steel are less for the same wall thickness. The difficulty of fabricating and welding plates more than a few centimeters thick limits the hull diameters, and added cross-sectional area is achieved in some designs by welding two typical hulls together to give a figure-eight cross section.

When the submarine is under way, diving vanes at the bow and stern, like seal's flippers, give some vertical maneuverability, but equilibrium is chiefly dependent on a careful adjustment of weight. A surface ship has great excess

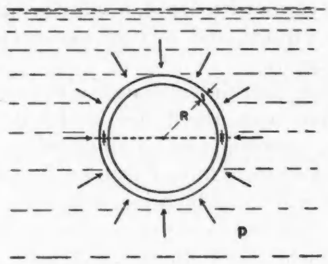


FIG. 2. Compression of a submarine hull.

buoyancy and can be said to be in very stable vertical equilibrium, though this technical statement is often questioned by a poor sailor. But a submerged submarine, though it no longer rolls or pitches, is in precarious equilibrium, and the nicest adjustment and distribution of weight is necessary to keep it at a fixed depth and on an even keel. Positive buoyancy for surfacing is obtained by the main ballast tanks, which lie like saddle bags along the outside of the pressure hull within a light-weight hull or fairing. These tanks are blown by compressed air, which forces the water out of valve openings at the bottom. They are flooded for submergence by opening vents at both top and bottom, and since they are always at sea pressure they do not require great structural strength. The trimming-tank system, which is quite separate, consists of tanks in the bow, stern and amidships and is used for making the small corrections for neutral buoyancy necessitated by the expenditure of fuel, stores or ammunition as well as for maintaining the proper buoyancy moment. Stability about the submarine's axis is not difficult to maintain because of the relatively short lever arm for rolling forces, but the length of the boat makes the tilt of the keel very sensitive to unequal weights in bow and stern. Since these trim tanks are carried partly full, they must be pressure proof and trimming is accomplished by pumping water in or out or from one tank to another.

The submarine is ideally operated at almost neutral buoyancy; but this may be a tricky condition to maintain, for both the volume of water displaced and its density may vary with depth of submergence. Both the submarine and the water are compressible and occupy smaller volumes at greater depths. Also, the sea tem-

perature varies with depth, usually being warmer near the surface, and thermal expansion affects the buoyancy.

Assuming that the submarine is in temperature equilibrium with the water at its depth, the balancing problem can be analyzed as follows. The net downward force F is $Mg - V\rho g$, where M is the mass of the submarine, V is the volume of water it displaces, ρ is the density of this water and g is the acceleration due to gravity. This net force must be zero for equilibrium, and for stable equilibrium an upward displacement must result in an increased downward force and *vice versa*. Taking the positive x axis downward in the direction of F and remembering that ρ and V are functions of x through their dependence on the pressure p and temperature T, we may take the first variation of F. Thus, since

$$F = Mg - \rho Vg \quad \text{and} \quad M = \rho V,$$

$$\delta F = -(\rho\delta V + V\delta\rho)g$$

$$= -\left[\rho\left(\frac{\partial V}{\partial p}\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{\partial V}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}x}\right) + V\left(\frac{\partial \rho}{\partial p}\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{\partial \rho}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}x}\right)\right]g\delta x$$

$$= -\left[(\alpha_w - \alpha_s)\frac{\mathrm{d}p}{\mathrm{d}x} - (\beta_w - \beta_s)\frac{\mathrm{d}T}{\mathrm{d}x}\right]Mg\delta x,$$

where

$$\alpha \equiv \frac{-1}{V}\frac{\partial V}{\partial p}, \quad \beta \equiv \frac{1}{V}\frac{\partial V}{\partial T},$$

and the subscripts $_w$ and $_s$ stand for water and submarine, respectively. For stable equilibrium the expression in brackets must be positive and, since $\mathrm{d}p/\mathrm{d}x \cong \rho g$,

$$(\alpha_w - \alpha_s)\rho g > (\beta_w - \beta_s)\mathrm{d}T/\mathrm{d}x \quad \text{(stability)}.$$

For zero temperature gradient this means that the compressibility of water must exceed that of the submarine. But this is practically never the case, for $\alpha_w \cong 5\times 10^{-11}$ dyne^{-1}, and α_s for a thin steel hull can be readily shown to be approximately equal to 5×10^{-10} dyne^{-1}. If the lower valves of the ballast tanks are open, water can enter the tank, compressing the air, and one has the very unstable equilibrium of the Cartesian diver. Thus, since $\beta_w > \beta_s$, $\mathrm{d}T/\mathrm{d}x$ must be negative and larger in absolute value than $\rho g(\alpha_w - \alpha_s)/(\beta_w - \beta_s)$. Using representative values—$\beta_w = 10^{-3}$ °C^{-1}, $\beta_s = 4\times 10^{-5}$ °C^{-1}, and the foregoing values of α_w and α_s—one finds that the temperature gradient for stable equilibrium must exceed about 5×10^{-2} °C per meter depth. Gradients of this magnitude occur rather frequently at depths of less than about 100 m, and submarines can float stably without being under way in these regions, which are known as "thermoclines" (Fig. 3). One further point should be mentioned, namely, that the density of sea water increases with its salinity. This can be very awkward for a submarine running in toward a river mouth, for it will find itself on the bottom if a sharp eye is not kept on the depth gage. As a result of these various factors which affect its buoyancy, a submarine is provided not only with pressure gages to indicate its depth but with recording thermometers and, occasionally, salinity meters as well.

Propulsion and Speed

The most important difference between the air and the water for automotive vehicles is that only the former supplies a component for the chemical reaction from which the energy for propulsion is derived. An airplane may have to use superchargers at high altitudes to compress the air sufficiently for the power plant, but as long as it remains in the atmosphere of the earth there is oxygen for burning its fuel. The submerged submarine, however, has no free oxygen available in the sea, and the suggestion occasionally made that the water be dissociated electrolytically and the resulting oxygen and hydrogen burned violates the principle of conservation of energy. In consequence the submarine has to be completely self-sufficient and must carry all the energy it needs for propulsion in some conveniently stowable form.

If the energy is carried in chemical form the fuel problem is the same as for any other vessel but the oxygen supply is different. Known technics offer two possibilities: compressed oxygen in tanks, or hydrogen peroxide. Both are expensive and to some extent hazardous. The cheap and convenient fuels are hydrocarbons, and therefore carbon dioxide is an exhaust

product that must be pumped out against sea pressure in great quantities with a serious loss in efficiency. Thus the internal-combustion engine for the submarine presents a serious problem. It has been partially solved by such power plants as the Walther engine, though the fuel cost is exhorbitant. The only competing method of energy storage as yet available is the electric secondary battery, which has every advantage over chemical energy storage except that the total energy available per unit weight or volume is much smaller. A battery is very efficient, involves little hazard when properly housed, produces no noise and little heat, has a relatively long life and produces mechanical energy by means of efficient motors. Batteries are the common source of power for all submarines when submerged; but in contrast with fuel oil, which has a heat of combustion of about 10 kw hr/kg, a battery can store only 0.01 kw hr/kg. Thus a submarine can cruise for months on the surface but can run submerged at a considerable speed for only a few hours. It then has to surface and charge the batteries, using its Diesel fuel supply, before it can again submerge. A high-temperature uranium reactor would appear to provide the ideal power plant for a submarine. The mass of fuel consumed is negligible and there is no exhaust problem, but at the moment there is no available power plant of this type either.

The density of water is the chief factor differentiating feasible velocities for submarines and aircraft. The velocities at which viscous forces are important are below those useful for submarines. When the velocities are comparable to those of sound in a medium, forces due to the compressibility, which give rise to wave motion, are of importance. This phenomenon is of concern for aircraft but not for submarines, as the velocity of sound in the ocean is about 1.5 km/sec. The intermediate region, where the retarding forces are due to turbulence and mass motion of the disturbed fluid, is the one relevant to submarines and subsonic aircraft. A retarding force having this origin clearly depends on fluid density, velocity and the linear dimensions of the moving vehicle (Fig. 4). Dimensionally, the force must be proportional to $a\rho v^2$, where a is a characteristic of the vehicle such as its frontal area, v is the vehicle velocity and ρ is the density of the fluid. Thus for vehicles of equivalent frontal area and streamlined for the mediums in which they operate, the retarding forces are proportional to the density and to the square of the velocity.

Comparison for air and water on a power basis is more illuminating. Writing P_a and P_w for the available power for an aircraft and submarine, respectively, and recalling that $P=Fv$, we have

$$P_a/P_w=(\rho_a v_a^3/\rho_w v_w^3)$$

or

$$v_a/v_w=(P_a\rho_w/P_w\rho_a)^{\frac{1}{3}}.$$

Thus an airplane with 8000-hp engines would be expected to go about 20 times as fast as a 1000-hp submarine operating in water ten times as dense as air. This represents about the observed ratio between a 300-mi/hr plane and a 15-knot submarine. It obviously represents a basic limitation on the order of magnitude of velocities achievable by submarines.

It is interesting to see that one source of retardation which affects surface ships need not retard submarines. This is the surface wave that sweeps away on either side from the surface vessel when it moves through the water. The rate at which energy is dissipated by the vessel in producing this wave can be calculated very approximately in the following way. Consider a standing-wave system which at the instant of maximum amplitude of motion is represented by Fig. 5. The fluid surface is instantaneously at rest in the assumed shape of a sine curve. The energy per unit width and length λ of the wave, where λ is the wavelength—this energy is then all in potential form—is given by the equation

$$\int_0^\lambda gy\,dx\cdot\tfrac{1}{2}y=\frac{1}{4\pi}\lambda g\rho \times\int_0^{2\pi} A'^2\sin^2\frac{2\pi x}{\lambda}\,d\left(\frac{2\pi x}{\lambda}\right)=\tfrac{1}{4}\rho g\lambda A'^2,$$

where A' is the amplitude of the standing wave. This may also be thought of as two progressive waves of amplitude $A[=\frac{1}{2}A']$ traveling in opposite directions. Therefore the energy per unit area of a progressive wave is $\frac{1}{2}\rho gA^2$. Thus if the ship wave is 1 m in amplitude, extends normal to the crest for about 5 m and is being

lengthened at the rate of 10 m/sec, the rate of doing work in producing the two waves on either side is about 500 kw, or about 700 hp. This is a very appreciable fraction of a small vessel's power and is important in retarding its motion through the water. However, the surface disturbance decreases exponentially with depth so that the wave resistance can be neglected if a submarine is well submerged. Thus if the boat is effectively streamlined it should be able to move faster for the same expenditure of power when submerged than when on the surface.

Submarine Communication

Possibly the greatest difference between ordinary and submarine physics results from the turbidity of ocean water and its opacity to electromagnetic waves. Most of our precise information ordinarily comes to us by vision, and the conditions for seeing appreciable distances are poor beneath the surface of the sea. The customary green-blue of the ocean shows that this is the region of greatest optical transparency, but even for it a range of a few hundred feet represents the maximum. A bright light can be perceived farther, but in the murkiness of the ocean all detail is lost and one is conscious only of illumination, as when seeing the sun through a dense fog.

Considering the greater wavelengths of the electromagnetic spectrum, the utility of radio and radar is immediately suggested. One would expect classical electromagnetic theory to be adequate for describing the propagation of these waves of comparatively low frequency. The conductivity σ of the sea is quite high, being of the order of 5 mho/m. The amplitude of an electromagnetic wave decreases exponentially as it

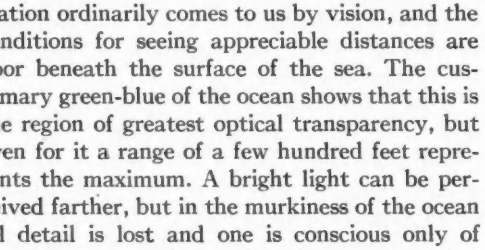


FIG. 3. Balancing submarine.

travels through a conducting medium, the power of ten being $14.6x(\sigma/\lambda_0)^{\frac{1}{2}}$, where x is the distance traveled and λ_0 is the vacuum wavelength, both expressed in meters. Thus in sea water the amplitude of the electric or magnetic field decreases by a factor of 10 for a distance of travel equal to 0.03 times the square root of the vacuum wavelength, when lengths are measured in meters (Fig. 6).

The exponential decrease is so predominant that simple conclusions can be drawn without particular reference to such factors as angle of incidence of the radiation on the water surface or coefficient of reflection there. A radio wave of long wavelength, say 10,000 m, decreases in amplitude by a factor of 10 for each 3 m of travel. Assuming that a signal a thousandth as intense as that above the surface could be detected by a submarine, its antenna could be used at a submergence of 9 m. It is important to note that this refers to detection and not to the use of such waves for anything akin to vision. Because of diffraction, radio waves could not be used to discern the contours of an object less than some hundreds of kilometers in extent. Radar waves could be used to delineate an object the size of a vessel, but their penetration in sea water is so small as to render them valueless.

For completeness, it is interesting to see whether electromagnetic waves of higher frequency than visible light could be used. In the x-ray, γ-ray and cosmic-ray regions the mechanisms of abstraction of energy from a beam of radiation are the quantum processes of photoionization, Compton scattering and electron-positron pair production. The absorption curves for high-energy radiation in water show that the absorption coefficient does not vary greatly but passes through a minimum at a few million electron-volts, but even the most penetrating radiation in this region is reduced exponentially to one-tenth intensity for every 15 cm of path. Thus these radiations can be of little use for signaling or communication in the sea.

If electromagnetic waves hold little promise for communication or the acquisition of information, the only alternative that suggests itself is sound. The properties of pure water are very favorable to the use of sound. Attenuation of a sound wave is much less in water than in air.

Heat conduction and viscosity do not appear adequate to account for the absorption coefficient that is observed, and the precise mechanism of loss of wave energy is still somewhat obscure.[1] However, the decrease in intensity of a plane sound wave in sea water is given approximately by the empirical expression $10^{-10^{-12}\nu^2 x}$ when the frequency ν is in the range from 50 to 50,000 c/sec, and x is measured in meters. Thus the sound wave drops to one-tenth intensity for $x=10^{12}\nu^{-2}$ m; for $\nu=100$ c/sec this is a very great distance, and even for $\nu=10^4$ c/sec, $x=10$ km. Thus attenuation is of little concern except for the highest frequencies, and the decrease in intensity of a sound wave in water is caused largely by the spreading out of the spherical wave from its source. The velocity of propagation of sound in water is about five times that in air, dispersion is in general negligible, and Doppler effects due to currents in the medium are small, because of both the higher sound velocity and the lower current velocities commonly encountered.

Acoustic Properties of the Sea

Though the use of sound would appear to afford the best means for transmitting information beneath the surface of the ocean, it is fraught with difficulties of all sorts and the submarine at present has to operate on the basis of much poorer intelligence than the airplane. For this reason it is probably just as well that it cannot move as rapidly. Some of the handicaps that are inherent in the use of submarine sound will be evident from a brief survey of the principal acoustic features of the sea.

The equation for the displacement ξ of a particle in one dimension in a perfectly elastic medium, which is the simple equation of motion of a plane sound wave, is

$$\partial^2\xi/\partial x^2=\rho\alpha\partial^2\xi/\partial t^2,$$

where t is the time, and the other symbols have their previous meanings. A solution of this equation is any function of the form $f(t\pm x/v)$, where $v=(\rho\alpha)^{-\frac{1}{2}}$; from the previously given values of these quantities v is about 1500 m/sec. This is much lower than the velocity of electromagnetic waves, and hence information about

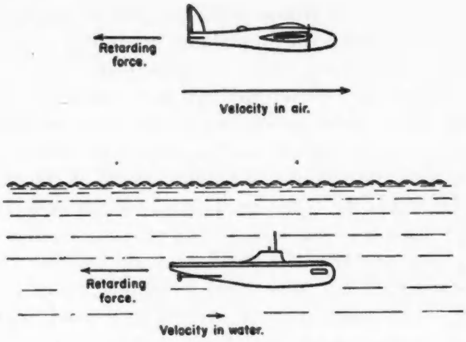


FIG. 4. Fluid resistance.

distant events is appreciably delayed. The equation of continuity,

$$p\alpha=\partial\xi/\partial x,$$

relates the pressure and displacement. The wave-pressure amplitude is limited approximately by the hydrostatic pressure because of the tendency to cavitation at negative pressure crests. This limits the intensity of a sound wave to $\frac{1}{2}(\alpha/\rho)^{\frac{1}{2}}p^2$. Near the surface of the water, where the pressure p is not much greater than atmospheric pressure, the intensity of a long harmonic wave cannot exceed about 1 w/cm². Thus the radiation from a feasible area such as 1000 cm² compares poorly with that from a powerful radio transmitter.

It is interesting to note that the ratio of pressure to particle velocity in a sound wave, which is known as the acoustic impedance, is given by $(\rho/\alpha)^{\frac{1}{2}}$, or ρv. For water this quantity is about 1.5×10^5 dyne sec/cm³, and for air it is about 36 dyne sec/cm³. Thus, in comparison with an acoustic wave in air, a wave in water is characterized by high pressures and small displacements. This completely alters the nature of submarine microphones and loud speakers, which are given the generic name *transducers*. Magnetostrictive and piezoelectric materials are used because they offer a good impedance match and hence high efficiency of transformation between electric and acoustic energy. For the same reason,

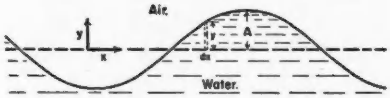


FIG. 5. Potential energy of a surface wave.

[1] See F. E. Fox and G. D. Rock, *Physical Rev.* **70**, 68 (1946), for a discussion of this interesting subject.

little sound is transmitted across the air-water surface, where the impedance match is so poor that only about 10^{-4} of the acoustic intensity incident from either medium is transmitted into the other. Thus the surface of the ocean is highly reflecting for sound waves, although, owing to the fluctuating and irregular nature of its contour, it can be considered as a specular reflector only for very long waves or for grazing incidence.

Near the ocean's surface small bubbles are common. They arise from breaking waves and other processes, particularly the motion of a ship in a choppy sea. These have an effect on sound transmission that is far out of proportion to the volume they occupy. Water containing a few small bubbles differs little in density from water itself; but, as the air in the bubbles is highly compressible, α is very much larger. In fact, the effective value of α is readily seen to be $\alpha_w(1+\alpha_a V_a/\alpha_w V_w)$, where the new subscripts $_a$ refer to air. Thus if there is, on the average, one bubble 1 mm in diameter in each cubic centimeter, this expression shows that the compressibility of water containing such bubbles is about $20\alpha_w$; here α_a, which is $1/p$, is taken at a depth of about 10 m. This would change the velocity of propagation by a factor of about 4.5 and cause an appreciable mismatch with the impedance of a transducer. Bubbles, animal and plant life, and larger fish all act as scattering centers, dispersing sound energy out of any well-defined beam. The ocean contains a great quantity of this scattering material, and some of the fish and crustaceans also produce sound themselves by beating on air bladders or snapping claws. It is clear that these various factors give rise to great confusion when sound is used for obtaining precise information under the sea.

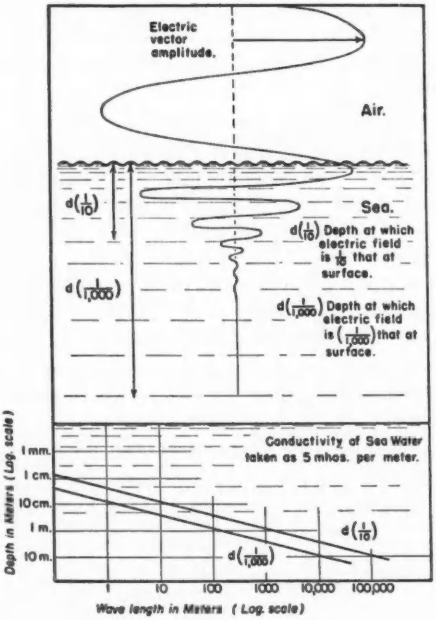


FIG. 6. Attenuation of an electromagnetic wave in the ocean.

Diffraction phenomena in the sea differ from those in the air only in a quantitative way. Since the velocity of sound is five times greater in water than in air, one must use five times the frequency to maintain the same ratio between the wavelength and the linear dimensions of transducers or other equipment. To a first approximation, the beam pattern of a transducer may be considered to be the same as the diffraction pattern formed when a plane wave passes through an opening similar in shape to that of the face of the transducer. Thus the intensity of the sound emitted or the sensitivity of reception decreases with increasing angle from the central axis, reaching a minimum at the angle θ given by $d \sin\theta = \lambda$ for a square opening of linear aperture d in the plane of θ. Minor maximums occur at greater angles. For a circular opening an additional numerical factor appears, and the beam width $\varphi[=2\theta]$ between first minimums on either side is given by $\varphi = 2 \sin^{-1} 1.22\lambda/d$ (Fig. 7). The reduction of λ to obtain added directionality implies the use of higher frequencies. When the audiofrequency range is exceeded the sound effects must be portrayed visually, as on a cathode-ray screen, or the sound may be heterodyned down to the audible range. These technics are well known, however, and impose no limitation on the use of high-frequency sound.

The refraction of sound rays due to the varying index of refraction is probably the most important phenomenon distorting the undersea picture as "viewed" by sound. The principal parameters upon which the velocity of sound depends are temperature, pressure and salinity. Salinity is of importance only near land where appreciable amounts of fresh water run into the

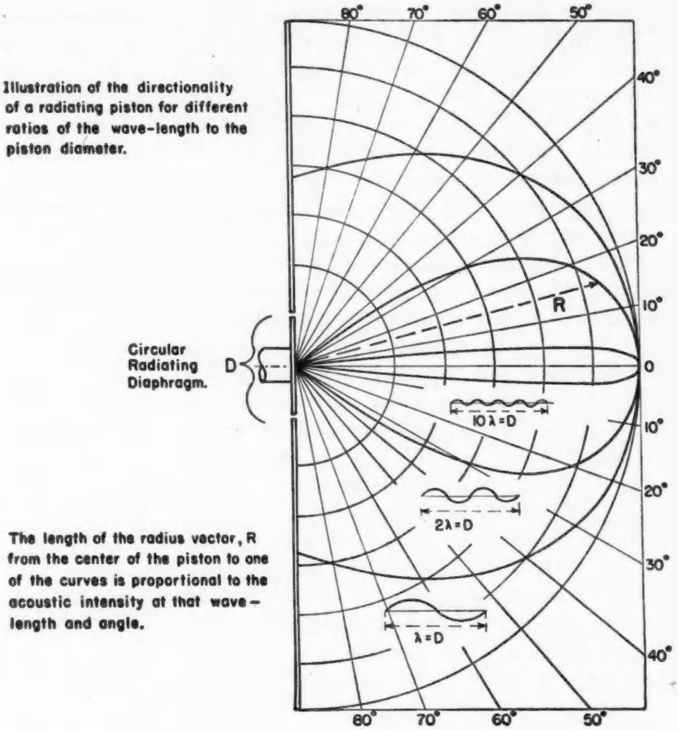


FIG. 7. Piston radiator beam patterns.

sea from rivers. In the deep ocean the temperature and pressure distributions determine the velocity and hence the refraction of sound. The effect of pressure is quite constant and therefore not particularly troublesome, but the temperature distribution in the first hundred meters below the surface is highly variable with time of year, time of day, cloudiness, wind velocity and other meteorological conditions. Except in or near pronounced ocean currents the temperature in general decreases monotonically but irregularly from a surface temperature of 10° or 20°C to that of maximum density (4°C) at great depths. The use of sound by surface vessels or by submarines in shallow water is greatly affected by the wide temperature variations and steep gradients that frequently exist over the upper 10 m of the ocean's surface. The gradients that permit a submarine to balance may also distort the pattern of sound waves to such an extent as to act as if regions of steep temperature gradient were almost impenetrable to sound signals.

The dependence of sound velocity on temperature and pressure is given for the temperature interval $6°C < t < 17°C$ by the expression, $v = 1410 + 4.21t - 0.037t^2 + 0.0175d$ (m/sec), where d (m) is the depth below the surface (Fig. 8). Two simple cases are of particular interest. The first is that near the surface for a uniform decrease in temperature with depth of a few tenths of a degree per meter. Taking the surface temperature as 15°C and the temperature gradient as γ °C/m, we have $v = v_s(1 - 2.12 \times 10^{-3}\gamma d)$, where v_s denotes the velocity of sound at the surface. For a medium in which the index of refraction varies in one direction, the path of a ray of sound leaving the surface at an angle θ_s can be calculated by Snell's law, which states that the product of the index of refraction and the cosine of the angle which a ray makes with the normal to the index gradient is constant. Thus $(\cos\theta)/v = (\cos\theta_s)/v_s$, or $(\cos\theta/\cos\theta_s) = (1 - 2.12 \times 10^{-3}\gamma d)$; and, by comparison with Fig. 9, the path of the ray is seen to be the arc of a circle leaving the

surface at the angle θ_s with a radius equal to $(2.12\times10^{-3}\gamma\cos\theta_s)^{-1}$. The rays are all bent downward in circular arcs a few thousand meters in radius for temperature gradients of a few tenths of a degree per meter. If the gradient is not uniform the curvature of the ray varies; and if the gradient fluctuates with time—as it does—the sound rays bend and weave, reminding one of vision through air over a hot plate.

The second case of interest is the one at great depths where the temperature decreases slowly toward that of maximum density of water and conditions are very stable and constant throughout the year. The result is not sensitive to particular assumptions so we may assume that the temperature increases quadratically from 4°C at 2000 m to 20°C at the surface, or $t=4[1+(2-d)^2]$, where the depth d is in kilometers. Inserting this value in the expressions for the velocity of sound one obtains, approximately, $v=1.46-17.5(2-d)+15.7(2-d)^2$ (km/sec). This expression has a minimum value for $d=d_m=1.45$ km, and, measuring height y vertically from this level, $v=1.45(1+1.1\times10^{-2}y^2)$. Since the velocity of sound increases for greater and lesser depths than d_m, rays tend to bend toward this level, and what is called the "deep sound channel" is formed (Fig. 10). Rays at a small inclination to the horizontal are constrained to remain in this channel, and, as attenuation can be practically neglected, the intensity falls off merely as the inverse first power instead of as the inverse second power of the distance. Sound signals from small explosive charges propagated in this channel have been heard across the ocean from Dakar to the Bahamas, taking about an hour for transit. The deep sound channel technic for getting fixes on a small explosion in the sound channel by measuring transit-time differences to several listening stations offers great promise for rescue work at sea. It is known by the acronym SOFAR (SOund Fixing And Ranging).

Snell's law can also be used to calculate ray paths in the sound channel for small angles of the propagation vector with the horizontal. To a satisfactory approximation, $\cos\theta=1-\frac{1}{2}\tan^2\theta$, and, as $\tan\theta=dy/dx$, the equation $\cos\theta/v=c$ becomes

$$dy/dx=\{[(1-cv_m)-cv_m\epsilon y^2]\}^{\frac{1}{2}},$$

where $c\equiv\cos\theta'/v'$, the primes indicating initial conditions, and $v=v_m(1+\epsilon y^2)$. This can be integrated immediately to yield

$$y=\left(\frac{1-cv_m}{cv_m\epsilon}\right)^{\frac{1}{2}}\sin[(2cv_m\epsilon)^{\frac{1}{2}}(x+\varphi)],$$

where φ is the constant of integration. If the initial displacement of the ray and its inclination

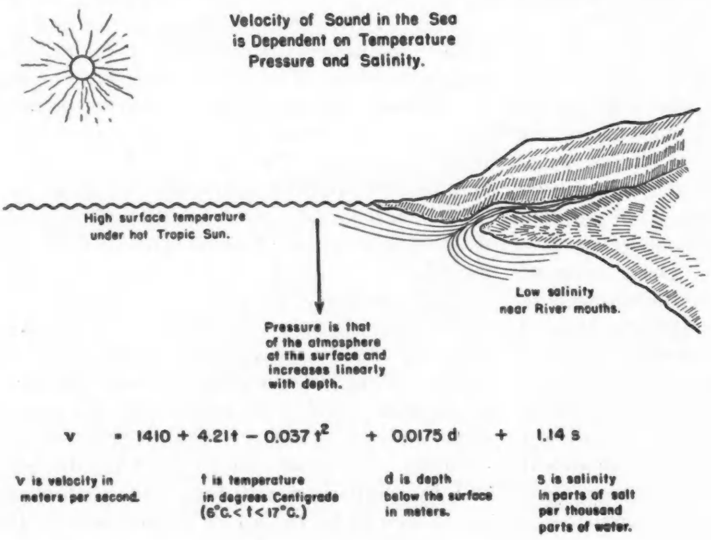


FIG. 8. Parameters affecting the velocity of sound in the sea.

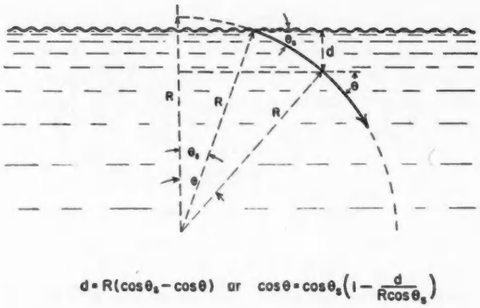


FIG. 9. Downward bending of sound in a linear temperature gradient.

with the axis are assumed to be small, the equation becomes approximately

$$y=\left(y'^2+\frac{\theta'^2}{2\epsilon}\right)^{\frac{1}{2}}\sin(2\epsilon)^{\frac{1}{2}}(x+\varphi).$$

Thus the trajectories are sine curves with the spatial period $2\pi/(2\epsilon)^{\frac{1}{2}}$, as shown in Fig. 11. Using the value of ϵ from the preceding paragraph, we find the wavelength to be 42.5 km. It is interesting to note that a ray coinciding with the horizontal at the depth of minimum velocity takes longer to travel a given distance than a ray that follows one of the sine curves with finite amplitude. The time taken to travel a path of length s is

$$\tau=\int_0^s\frac{ds}{v}=\int_0^{2\pi/(2\epsilon)}\frac{dx}{cv_m(1+\epsilon y^2)^2}$$
$$=\frac{2\pi}{(2\epsilon)^{\frac{1}{2}}v_m}\left[1-\tfrac{1}{2}(\epsilon y'^2+\tfrac{1}{2}\theta'^2)\right].$$

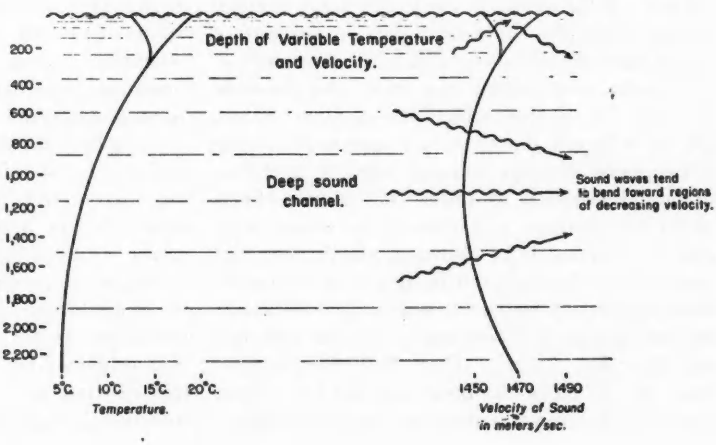
Thus if both y' and θ' do not vanish, the sound ray has a shorter travel time than if they do. This is clearly seen in a sofar signal, which rumbles in for about 10 sec, ending with a sharp clap as the slowest ray along the central channel finally arrives.

Use of Sound by Submarines

The physical properties of the sea determine the nature of SONAR (acronym for SOund Navigation And Ranging), which is the technic of submarine sound. It is clear from the preceding section that the sea is a very difficult medium in which to work. It is full of sources of sound that are generally distracting, such as fish and surf noises, and the most serious of all, contains the sound made by the ship itself. The latter occurs just where it does the most harm, right in the neighborhood of the transducer. If the ship is noisy it is almost impossible to use sound effectively, as the low-level signals are completely obscured by the background noise. The first rule for sonar is to quiet the ship as much as possible. Some of the naturally occurring noises, such as those produced by surf and crustaceans, are useful in locating shores and shallows; but in general the ocean sounds are of value only for concealing one's own noise from others.

Attenuation of sound is small, and the sea is full of scattering objects and bounded by an undulating, reflecting upper surface and an irregular bottom of varying reflecting character. Moreover, the sound paths bend and contort as they travel through regions of fluctuating tem-

FIG. 10. Formation of the deep sound channel.



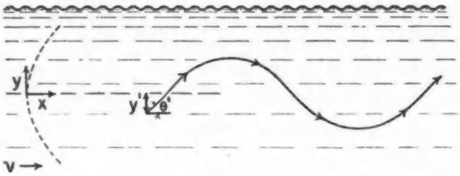


FIG. 11. A ray in the sound channel.

perature. On the whole it is surprising that any effective use can be made of sound at all, for a crude optical analog would be the use of vision in a room walled with undulating mirrors and filled with turbulent wisps of steam and points of light. However, a program of basic study of the properties of the ocean and of painstaking design, development and test of equipment has evolved sonar gear capable of serving the most urgent needs of submarines. Most of the practical problems to be solved are electronic and engineering ones and hence beyond the scope of submarine physics. However, this account would be incomplete without a general description of listening and echo ranging, which are the two broad technics for the use of sound in the sea.

Listening, as the name implies, is the passive reception of sounds reaching the submarine from some sonorous object. The intensity of the sound as heard by the submarine is inversely proportional to the square of the distance traveled, modified by the particular refraction conditions encountered. Also, since high frequencies are attenuated appreciably, the low frequencies are relatively accentuated and the harmonics suppressed. If the signal at the hydrophone is much weaker than the ambient noise, no amount of subsequent electrical amplification can ever derive useful information from it. In the constant striving for greater acuity the primary efforts are to reduce ambient noise background at the hydrophone and to increase angular and frequency selectivity in order that sounds from unwanted quarters and of undesired frequencies can be excluded from audition. Our earlier discussion of diffraction and beam patterns showed that angular and frequency selectivity are closely related; low frequencies imply broader patterns and less directionality than high frequencies. Also, the width of the band selected by a filter cannot be reduced too much for general listening,

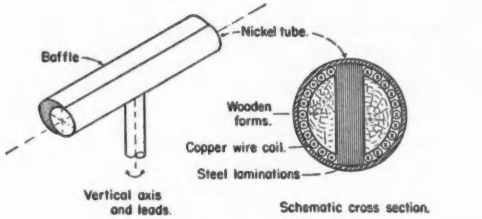


FIG. 12. Illustrative magnetostrictive hydrophone (JP).

or much of the character of the sound will be lost and useful information sacrificed.

The JP magnetostrictive hydrophone is representative of a sensitive and rugged listening device (Fig. 12). It consists of a sealed nickel tube with a central septum of steel laminations about which a coil of wire is wound on the two halves of a wooden form. The nickel, acting as an armature between the edges of the steel laminations, is magnetized by setting up a current in the coil. Thereafter, changes in pressure on the tube wall change the remanent flux through the coil because of the magnetostrictive property of nickel. Sound waves are thus converted into an alternating potential difference between the coil terminals, which can subsequently be amplified, filtered and detected. The tube is about 1.5 m long, so the half-angular width of the central beam is given to within a few percent by $\lambda/1.5$ for wavelengths less than about 0.75 m. At 15,000 c/sec the half-width is about 3°, and at 2000 c/sec it is about 30°. The nickel tube is only a few centimeters in diameter, so there is practically no directionality in the plane normal to its axis, except that a baffle suppresses response from the back. The beam pattern thus resembles a segment of an orange, and rotation about an axis perpendicular to that of the tube permits azimuthal listening search.

In echo ranging a short burst of sound is emitted by sending a pulse of high-frequency current through the transducer; thereafter the terminals are automatically switched to a receiver, and the transducer is used as a listening hydrophone to detect any echo that may be returned. Echo-ranging transducer design is much the same as for listening transducers, except that echo-ranging transducers must be efficient transmitting devices up to cavitation power. Additional listening selectivity, however, can be

achieved by virtue of the fact that only a narrow frequency band, namely that transmitted, need be selected and amplified. This permits the exclusion of a considerable amount of unwanted noise. The range of the object returning the echo is given immediately by $\tau/2v$, where τ is the time interval between transmission and reception and v is the velocity of sound. The factor 2 arises because the sound passes each way during the time τ.

Figure 13 is a schematic depiction of typical transducer construction for echo ranging. A large number of nickel tubes are welded to the back of a steel slab, which has a narrow flange for mounting on the ship's hull or other supporting structure. The lengths of the tubes and thickness of the slab are such that the combination resonates to the chosen frequency, with the front face of the slab and free ends of the tubes as loops and the supporting flange as a node. A pulse of high-frequency electric current passes through the coils surrounding the nickel tubes, and the magnetostrictive effect sets the system into oscillation. The impedance match between the slab and the water permits the radiation of sound energy in that direction, and the poor match with the air on the tube side prevents loss of acoustic energy there. Similarly, a wave incident on the slab from the water sets the system into oscillation, and the emf generated in the coils is amplified and detected by a receiver.

One of the first applications of this technic was to the echo depth sounder, or *fathometer*. In this device the transducer is faired into the ship's bottom with a broad beam directed vertically downward. The echo interval is a direct measure of the depth of the bottom of the ocean below the keel. Automatic circuits key the transducer periodically, and depth is read from a dial or recording tape directly in fathoms or meters.

The bottom echo is usually easy to observe,

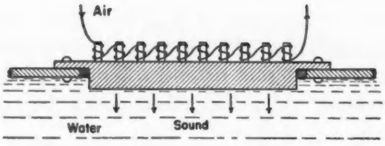


FIG. 13. Schematic transducer.

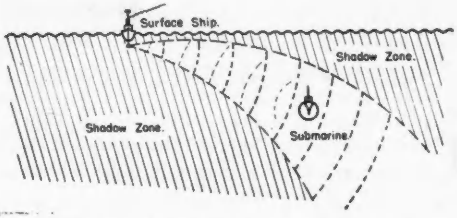


FIG. 14. Echo ranging on a submarine.

but the technic can also be used to detect and locate much smaller objects which return correspondingly fainter echoes if conditions are not too unfavorable. Hulls of surface ships and submarines, whales and schools of fish, beds of kelp and regions of sharp discontinuities in temperature or salinity all return echoes of sorts. A typical echo-ranging transducer for submarine search is mounted with the normal to its face horizontal (Fig. 14), and provision is made to rotate it about a vertical axis for azimuth search. It resembles an acoustic searchlight that can be trained in azimuth and sometimes in elevation, and if stabilized against the ship's motion it can yield sufficiently precise information on the angular bearing of an echoing object. Range is derived from elapsed time just as with the fathometer. The difficulties inherent in target identification will be obvious. However, skilled operators can infer much from the character of the echo. If there is relative motion along the line joining ship and target the Doppler effect, enhanced by heterodyning, is frequently a help in identification, though whales are even more mobile than submarines. The determination of the depth of the echoing object is greatly handicapped by proximity to the surface, which contributes reflections, and by the bending of the sound beam under the influence of changing temperature conditions.

Physics and engineering have done much for sonar, but more beds of kelp and schools of fish than submarines have been depth charged in the long history of the Navy's operations. Nevertheless, sonar is the technic upon which reliance for submarine intelligence must rest, and the acuity, persistence, alertness and skill of the sonar operator can accomplish wonders with the best modern equipment, even in a medium as intractable as the ocean.

Magnetization and Rotation*

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THE idea of a causative relation between the rotation of a body and its magnetization, parallel to the axis of rotation, first emerged in connection with cosmical magnetism 350 years ago, when William Gilbert attributed the rotation of the earth at least chiefly to its magnetism.

This idea never made any headway; but its converse, for many reasons, began to develop in the second part of the last century. Nearly 60 years ago, Bigelow[1] in this country, and a little later Schuster[2] in England, from the general appearance of the polar streamers in the solar corona at sun-spot minima inferred that the sun, like the earth, was probably magnetized along its axis of rotation. Much more extensive and precise work on the coronal streamers, by W. W. Campbell and collaborators,[3] in 1925 were likewise favorable to this hypothesis.

In the meantime, Hale and his collaborators at Mount Wilson actually proved that the sun was magnetized, the polarity being related to the direction of rotation in much the same way as in the case of the earth. And recently, H. W. Babcock[4] has made the discovery that a number of the other fixed stars also are magnetized along their axes of rotation.

Although many hypotheses have been advanced that are qualitatively consistent with these phenomena, there is still no theory based on known facts which has succeeded in explaining more than a minute part of their magnitudes. I shall return to this subject at the end of this paper.

The work of Oersted, Ampère, and their contemporaries early in the last century showed that the circulation of electricity in a coil of wire produces an axial magnetic field; and Faraday showed that, conversely, the creation of an axial magnetic field produces circulation of electricity in the wire. Ampère and Weber showed it to be highly probable that currents of electricity in molecules or molecular paths—producing axial magnetic fields, or being produced by changing axial fields—were fundamentally responsible for gross magnetism and diamagnetism.

The experiments of Rowland, in which electrified disks in motion produced magnetic fields similar to those produced by currents in wires, and the much later converse experiments of V. Crémieu[5] (when properly interpreted) gave further evidence of the relation we are considering. And in more recent years we have gained a vast amount of evidence for a similar relation for electrons, protons, etc., for atomic nuclei, and even for neutrons—all revolving in orbits or spinning about an axis, and all having magnetic moments.

In the half-hour at my disposal it is impossible to do more than mention all this work and also to discuss, even very briefly, two other groups of phenomena, related to them, in the investigation of which I have long been particularly interested, and with which this paper is chiefly concerned. These are: gross dynamical or electrical phenomena which must be attributed to the inertia of the free ions in conductors or bound ions in insulators; gross dynamical or magnetic phenomena which are due to the behavior of the elementary magnet as a gyroscope, and which are usually known as gyromagnetic phenomena.

I. PHENOMENA DUE TO INERTIA OF IONS

In this group four kinds of experiments have been made.

1. Current Produced by Acceleration

If a wire is accelerated in the direction of its length, the free electricity will be differently accelerated, lagging behind when the wire speed is increased and going ahead when the speed is

* A paper presented by invitation of the American Physical Society at its 283rd meeting, Jan. 2, 1948, at Los Angeles, California.

[1] F. H. Bigelow, *Smithsonian Institution Rept.* (1889).

[2] A. Schuster, *Brit. Assoc. Advancement Sci., Rept.* (1891).

[3] *Proc. Am. Phil. Soc.* (1925).

[4] H. W. Babcock, *Astrophysical J.*, **105**, 105 (1947); *Pubs. Astron. Soc. Pacific* (1947).

[5] V. Crémieu, *Soc. Trans. Phys. Bull.* 156, 2 (1901) and H. A. Wilson, *Phil. Mag.* **2**, 144 (1901).

decreased. Thus the acceleration of the wire gives rise to an electric current in it and hence, if the wire is formed into a coil, to a magnetic moment. This effect was described by Maxwell,[6] and was apparently looked for by him in or about 1861.

(*a*) *Tolman and collaborators.*—This effect in metallic conductors was first observed by Tolman and Stewart at Berkeley in 1916 and has since been further investigated by Tolman and collaborators in different ways.[7] Their investigations serve to determine the sign of the electric carrier and the ratio of its mass m to its charge e. They show that the sign of the charge is negative for the substances investigated—copper, silver and aluminum. The mean values of m/e ranged in the different investigations from about 15 percent below the standard value for an electron to about 19 percent above.

(*b*) *Colley.*—If an electrolyte instead of a metallic conductor is accelerated, a phenomenon of the same general nature occurs, on account of the difference between the masses and mobilities of the ions. Successful, but not precise, experiments of this sort were made by Colley,[8] not long after Maxwell.

2. Acceleration Produced by Current

(*a*) *Barnett.*—If the aforementioned coil is free to move about its axis and a current in it is altered, the free electricity will be accelerated and the coil itself will be accelerated in the opposite direction, their changes in angular momenta being equal in magnitude and opposite in sign. This effect also was apparently looked for by Maxwell,[6] and it was looked for by Oliver Lodge in or before 1892;[9] but it was first observed by the author,[10] with the help of William Arnold, in 1930. This investigation gave for m/e in copper, the only substance investigated, the standard value for an electron within 3 percent, with an experimental error of the same magnitude. This work, which was greatly retarded by the war, is being extended.

(*b*) *Kettering and Scott.*—My own experiments have recently been repeated, in a modified form, and with greatly increased precision, by Kettering and Scott.[11] For aluminum and copper they find for m/e the standard value with an error of only about 0.25 percent.

(*c*) *Kikoin and Goobar.*—A third investigation of the same effect, on lead in the superconducting state, was made by Kikoin and Goobar[12] in 1938–40. The current did not traverse a coil, as in the other investigations, but was induced on the surface of a small sphere suspended at the center of a fixed magnetizing coil. The standard value of m/e was obtained with an error of 3 percent.

3. Centrifugal Action

(*a*) *Des Coudres; Tolman.*—The first experiments of the centrifugal type were made by Des Coudres,[13] on electrolytes, in 1893. Further experiments of the same general kind, but much more precise, were made by Tolman[14] in 1910. From measurements of the potential differences due to the centrifugal separation of the ions, the transport numbers were calculated, but no attempt was made to measure the magnetic effects.

(*b*) *Nichols.*—This was the case also in an investigation by E. F. Nichols[15] in 1906. An aluminum disk was rotated rapidly, and an attempt was made to measure the potential difference developed between two brushes near the center and the edge. It was assumed that the positive ions remained radially fixed while the electrons were driven out centrifugally until kinetic equilibrium was reached. For the standard value of m/e a potential difference of about 10^{-8} v from center to edge should have been developed. Actually, irregular potential differences 500 times as great were obtained.

(*c*) *Lébedèw.*—A few years after Nichols, in 1912, experiments were begun by Lébedèw[16] to see whether terrestrial magnetism could be accounted for by centrifugal action. In this work small toroidal rings of ebonite, brass, water and

[6] J. C. Maxwell, *Electricity and magnetism*, secs. 574, 575, 577.
[7] *Physical Rev.* **9**, 164 (1917); **21**, 525 (1923); **28**, 794 (1926).
[8] R. Colley, *Wied. Ann.* **17**, 56 (1882).
[9] O. J. Lodge, *Modern views of electricity* (1892), p. 97.
[10] S. J. Barnett, *Phil. Mag.* **42**, 349 (1931).
[11] Kettering and Scott, *Physical Rev.* **66**, 257 (1944).
[12] Kikoin and Goobar, *Compt. rend. acad. sci. USSR*, **19**, 249 (1938) and *J. of Phys. USSR* **3**, Nos. 4 & 5 (1900).
[13] Th. Des Coudres, *Wied. Ann.* **49**, 294 (1893).
[14] R. C. Tolman, *Proc. Am. Acad. Arts Sci.* **46**, 109 (1910).
[15] E. F. Nichols, *Physik. Z.* **7**, 640 (1906).
[16] P. Lébedèw, *Ann. Physik.* **39**, 840 (1912).

benzol, all non-magnetic, were rotated about their axes at a frequency of about 500 rev/sec. Lébedèw developed a centrifugal formula from which he calculated that if the earth were made of material with properties similar to those of his toroids he should have obtained a magnetic intensity equal to about one one-hundredth part of the earth's equatorial intensity. He could have detected one ten-thousandth part, but in no case was any appreciable effect observed. The experiments were cut short by Lébedèw's death.

(*d*) *Swann and Longacre.*—Somewhat similar experiments, with a sphere of copper 8 in. in diameter rotating about 200 rev/sec, all it would stand, were made by Swann and Longacre[17] in 1928, but again with results too minute to be significant. I shall return to this experiment later.

4. Gyroscopic Effect

Another type of electron inertia experiment was made by Maxwell,[6] again in or about 1861. If the wheel of a gyroscope is spun rapidly about a horizontal axis and the frame carrying the wheel then rotated slowly about a vertical axis, the wheel tips up or down so as to make the direction of its rotation coincide more nearly with the direction of the impressed rotation about the vertical. If now the gyroscope wheel *in motion* is replaced by a cylindrical coil *at rest*, but traversed by a steady electric current, and if the electricity in motion *in the wire* has all the same sign, the coil will have *concealed* angular momentum and should behave exactly like the gyroscope wheel when rotated about an axis perpendicular to its axis of symmetry.

It was essentially this experiment that was tried by Maxwell, though with considerably different apparatus. No appreciable effect was observed, undoubtedly because the extraneous disturbances due to the earth's magnetic field and the mechanical imperfections of the apparatus were too great, and the method of observation too insensitive.

II. GYROMAGNETIC EFFECTS

The experiments hitherto considered are pure ion-inertia experiments, and do not involve the assumption of any *magnetic* quality associated with the ion itself or with its motion in an orbit. We now come to gyromagnetic effects *proper*, which are gross magnetic or dynamical phenomena due to the behavior of the elementary magnet as the rotor of a gyroscope.

(*1*) *Maxwell.*—According to the hypothesis of Ampère, as everybody knows, each of the elementary magnets of which a magnetic body is, at least in part, built up, is constituted of a minute electrical whirl endowed with angular momentum, and must thus have the mechanical properties of our gyroscope wheel.

Since the magnetic axis of every whirl bears the same relation to its rotary momentum, an unmagnetized rod, in which the axes of the whirls point in all directions equally, will have no resultant momentum about any direction, just as it has no resultant magnetic moment. If the rod is magnetized to saturation, however, the magnetic axes of all the whirls point in the same direction, and thus the rod will have a resultant angular momentum and should exhibit the properties of a gyroscope.

If, therefore, we replace our wheel by a highly magnetized rod of iron, or by an electromagnet, and then rotate the framework about the vertical, as before, the axis of the magnet should tip up or down according to the direction of rotation about the vertical—provided extraneous disturbances are not too great to mask the effect.

This experiment was made by Maxwell at the same time as that on electron inertia just described, an electromagnet, from which the iron could be removed at will, being used for both. In neither case was any appreciable effect obtained. Maxwell concluded that if either a magnet or a coil of wire carrying an electric current contains matter in motion, the momentum of this matter must be very small in comparison with any quantity which he could measure.

(*2, a*) *Barnett;*[18] *magnetization by rotation.*—It does not seem to have occurred to Maxwell to make an experiment in which every one of the countless multitude of magnetic elements in a magnetic body should simultaneously replace his magnet, and to measure the *gross change* in the orientation of all these elements by a *magnetic* method.

[17] Swann and Longacre, *J. Franklin Inst.* **206**, 421 (1928).

[18] S. J. Barnett, *Physical Rev.* **6**, 239 (1915).

The first experiment based upon this idea appears to be one made more than 50 years ago by John Perry, who tried, but without success, to detect magnetization in an iron rod produced by rotating it. In 1909, the same idea occurred to the author, who, with the help of L. J. H. Barnett, then began experiments which were first completely successful in 1914, when they were presented to the Ohio Academy of Sciences and to the American Physical Society. These were the first successful experiments in the whole field of gyromagnetic phenomena, and have been fully confirmed by much later work.

If it is assumed that the magnetic elements in a body are all alike, the ratio of angular momentum to magnetic moment of each being ρ, the *gyromagnetic ratio*, simple theory shows that the rotation of the body about any axis with the angular velocity Ω (rad/sec) will produce at any point within it exactly the same intensity of magnetization as a uniform axial field with magnetic intensity

$$H=\rho\Omega. \qquad (1)$$

Several experimental methods are available. In each we find the deflection of a measuring instrument caused *solely* by rotation, at a known speed Ω, and also the magnetic field intensity H necessary to produce, *alone*, the same deflection (or we simultaneously compensate one by the other). Then we have

$$\rho=H/\Omega. \qquad (2)$$

The earliest completely successful investigations, in 1914 and 1915, were made by a method of electromagnetic induction. The numerical results of 1914 give for the gyromagnetic ratio the value $\rho=1.02m/e$. The somewhat better results obtained in 1915 by the same method, with improvements, give $0.96m/e$. Within the limits of the experimental error, both results are only one-half the value, namely, $2m/e$, to be expected on the basis of the electron orbit as magnetic element.

The investigation gave a direct proof, and the first proof, of the actual existence in iron of the molecular currents of Ampère, theretofore hypothetical; it proved that the electricity in these currents is negative, and has mass, or inertia; and it revealed a second and entirely new method of magnetizing bodies.

The magnetization produced, however, by speeds experimentally attainable is exceedingly minute. Thus, as the insertion in Eq. (2) would show, rotating a body at 100 rev/sec is equivalent to putting it in a magnetic field which is only about 1/15000 as intense as the earth's field in Los Angeles. The effect is in the right direction to account for the magnetization of the earth, but it is about 10 billion times too small. However, it is nearly one billion times as great as any other direct rotation effect which has been proposed for this purpose.

One of the most important results of the investigation is the magnitude obtained for the gyromagnetic ratio, which, as previously stated, is about one-half the value calculated for an electron moving in an orbit. This discrepancy was long known as the *gyromagnetic*, or *magneto-mechanical*, anomaly; and the result indicates that the magnetic element consists *primarily* of a Lorentz electron—that is, a spherical electron whose charge is uniformly distributed over its surface—spinning on a diameter, and not of an electron moving in an orbit. More precise investigations in my laboratories made later show, however, that in most cases the orbital motion also contributes, and to an extent which varies from substance to substance, as we shall see later.

(*2, b, c, d*).—Three additional investigations of this effect have been made, the first two by magnetometer methods, the last by a method of electromagnetic induction. The first, by comparatively rough experiments, showed that iron, cobalt and nickel have nearly identical gyromagnetic ratios. The second (1920–25)[19] and third (1939–44)[20] were very elaborate, and gave concordant and relatively precise determinations for many ferromagnetic substances, the mean values of the gyromagnetic ratios ranging from about $1.00m/e$ for Heusler alloy to something more than $1.07m/e$ for cobalt and a cobalt-nickel alloy. This last investigation, greatly retarded by the war, is still going on, with improvements. Preparations are now under way for a systematic study of cobalt-iron, cobalt-nickel and iron-nickel alloys. Summarized results are given in Table I.

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[19] S. J. and L. J. H. Barnett, *Proc. Am. Acad. Arts Sci.* **60**, 125 (1925).

[20] S. J. Barnett, *Proc. Am. Acad. Arts Sci.* **75**, 109 (1944); *Physical Rev.* **66**, 224 (1944).

TABLE I. Values of gyromagnetic ratio ρ of ferromagnetic substances; xe/m from the three extensive investigations in the author's laboratories ($e/m = 1.759 \times 10^7$ emu).

Investigation	Effect investigated		Material investigated: Soft iron	Steel	Nickel	Hipernik	Hopk. alloy	Permalloy	Cobalt	Cobalt iron	Cobalt nickel	Heusler alloy
I	Barnett	$(\rho e/m)_{\mathrm{I}} = A$	1.049	1.047	1.031	—	1.016	1.054	1.070	1.067	1.068	1.011
II	Einstein-de Haas	$(\rho e/m)_{\mathrm{II}} = B$	1.032	1.038	1.051	1.051	1.023	1.046	1.085	1.025	1.076	—
III	Barnett	$(\rho e/m)_{\mathrm{III}} = C$	1.028	1.039	1.054	—	1.019	1.053	1.072	1.029	1.080	0.989
I and III	Barnett	$\frac{1}{2}(A+C)$	1.038 ±0.010	1.043 ±0.004	1.042 ±0.010	—	1.018 ±0.002	1.054 ±0.000	1.071 ±0.001	1.048 ±0.019	1.074 ±0.006	1.000 ±0.011
I, II and III	Both	$\frac{1}{2}[B+\frac{1}{2}(A+C)]$	1.035 ±0.003	1.040 ±0.002	1.046 ±0.004	—	1.017 ±0.001	1.050 ±0.004	1.078 ±0.007	1.036 ±0.012	1.075 ±0.001	—

(3) *Einstein-de Haas effect.*—The effect converse to mine, namely, gyromagnetic rotation by magnetization, was first looked for by O. W. Richardson,[21] in 1907, but the investigation was not completed. The effect was first obtained by Einstein and de Haas[22]—the rough magnitude (ca. $2m/e$ instead of *ca.* m/e) in 1915, the sign (negative) in 1916.[23] Many other investigations have since been made.

In most experiments on this effect a circular cylinder, or *rotor*, of the substance under investigation is suspended with its axis vertical in a fixed framework. Surrounding the cylinder coaxially is a magnetizing coil of insulated wire. The motion of the cylinder produced by changes in axial magnetization is studied.

When the axial magnetic moment μ changes at the rate $\dot{\mu}$, the concealed angular momentum $M[=\rho\mu]$ changes at the rate $\dot{M}[=\rho\dot{\mu}]$; hence the angular momentum J of the rotor and coil together will change at the rate

$$g = \dot{J} = -\dot{M} = -\rho\dot{\mu}, \tag{3}$$

which is the gyromagnetic torque on the rotor and magnetizing coil *together*. In most work it has been assumed, as is now known to be true, that the torque on the magnetizing coil vanishes. The frictional torque is taken (consistently with experiment) as proportional to the angular velocity of the rotor.

If disturbing torques, also producing changes in J, are present, Eq. (3) will, of course, not be valid. All of these can be eliminated or corrected for by suitable methods.

The process by which the equal and opposite momenta are produced is doubtless somewhat as follows. For definiteness, assume the rotating electricity in the elements to be positive, and assume the field to be impressed in the positive direction along the axis of the rod. Then the application of the field (or its increase) will produce a torque on each element which makes it precess around the direction of the field with a *negative* angular velocity. Interaction with the adjacent elements (retarding the precession) thus produces a *negative* axial torque upon the rest of the body and a *positive* axial torque upon the particular element under consideration. This latter torque increases the degree of alinement of the axis of spin (and magnetic moment) of the element with the direction of the field. In this way, as the magnetic field intensity increases, the total axial momentum of the elements and the axial momentum of the rod increase together in opposite directions.

Two general methods of investigating the effect have succeeded—ballistic methods and resonance methods. For ballistic methods Eq. (3) is altered to the integral form

$$\delta J = \int g\mathrm{d}t = -\rho\delta\mu, \tag{4}$$

the rotor being initially at rest. That is, the change δJ in the angular momentum of the rotor is measured for a given change $\delta\mu$ in the magnetic moment produced by altering the current in the magnetizing coil. The quantity $-\rho$ is then calculated as $\delta J/\delta\mu$. The quantity δJ is obtained from

[21] O. W. Richardson, *Physical Rev.* **26**, 248 (1908).
[22] Einstein and de Haas, *Verhand. deut. Physik. Ges.* **17**, 152 (1915).
[23] Einstein, *Verhand. deut. Physik. Ges.* **18**, 173 (1916); de Haas, *Verhand. deut. Physik. Ges.* **18**, 423 (1916).

the experiments by using the standard ballistic formula.

For resonance methods, Eq. (3) is used directly in the differential form, $-\rho$ being determined in effect as the ratio $g/\dot{\mu}$.

The experiments in my own laboratory[24] on ferromagnetic substances are much the most elaborate, and the most precise, that have been made on this effect. They are, in general, quite concordant with the results obtained on the converse effect and of greater precision. The results obtained in my own laboratories since the precise work began are summarized in Table I, together with those on the Barnett effect. The close agreement between the results of investigations III and II is apparent, also the generally good agreement between the results of I and II.

The accurate calculation of the excess over m/e to be expected for the gyromagnetic ratio for a given ferromagnetic substance is very difficult, and has never been made. That the values obtained in my experiments are of the general magnitude to be expected, however, has been concluded by Gorter and Kahn, Van Vleck, Kramers, and others.[25]

For the ferromagnetic material pyrrhotite, Coeterier and Scherrer[26] have obtained very nearly $3m/e$ for ρ, which can be explained, though very uncertainly, on the assumption that the magnetic element is a *d* electron with its orbital momentum oriented antiparallel to its electron spin.

Sucksmith[27] has made a series of determinations for paramagnetic substances, and in most cases has obtained gyromagnetic ratios that are in agreement with the values calculated from spectroscopic data on the theories of Van Vleck and Stoner.

III. GYROMAGNETIC EFFECTS NOT INVOLVING THE ROTATION OF GROSS BODIES

In addition to the gyromagnetic experiments which have now been described, and which involve the actual rotation of *gross bodies*, a number of others have been made which are gyromagnetic in their nature, but involve only molecular motion. Only one of these can be discussed here in even slight detail, namely, what I have called *magnetization by rotary fields.*

In experiments on my own effect, magnetization occurs not because the whole rod rotates, but because the rotation of the rod entrains the magnetic elements and causes *them* to rotate. Hence the idea arose that if a rod were placed in a cross magnetic field due to a two-phase electric system and the magnetization vector were thus rotated, longitudinal magnetization should result. In 1922–24 experiments of this sort were made in O. W. Richardson's laboratory by J. W. Fisher,[28] who expected to get longitudinal magnetization equal to that which would have been obtained from my own effect at the same frequency. No effect, however, was obtained.

It was shown by the author in 1925, however, on the hypothesis most favorable to an explanation of the results by rotation of the elements, that Fisher's expectation of the magnetization which would result should be multiplied by a small factor equal approximately to 1.5 times the ratio of the cross-magnetization to the saturation magnetization of the material. Fisher's combined errors were of the same order as this reduced result.

The expectation on the rotation hypothesis should probably be reduced much farther because the torques opposing axial alinement of the elements are probably much greater when the elements rotate with respect to the remaining atomic structure than when they and the rest of the structure all rotate together, as Epstein has suggested. In the calculation referred to the torques opposing alinement were assumed to be identical in the two cases.

Moreover, as is well known, there are indications that in the early part of the magnetizing process, that is, in weak fields, the magnetization proceeds by quantum jumps, or reversals, of the elements, and *not* by changes in their *orientation*, as required by the classical theories; and this, as suggested to me by Einstein, would require a null effect in these experiments.

[24] See especially S. J. Barnett, *Proc. Am. Acad. Arts Sci.* **73**, 401 (1940).

[25] Gorter and Kahn, *Physica* **7**, 753 (1940); C. J. Gorter, *Physical Rev.* **60**, 836 (1941), and "Le Magnétisme" (*Reports of the Strasburg réunion sur le magnétisme* vol. II, 1940).

[26] See *Helv. Physica Acta.* **8**, 52 (1935); also D. R. Inglis, *Physical Rev.* **45**, 118 (1934).

[27] W. Sucksmith, *Proc. Roy. Soc.* (*London*) (A) **128**, 133, 135 (1930–32).

[28] J. W. Fisher, *Proc. Roy. Soc.* (*London*) (A) **109**, 7 (1925).

An investigation by the author,[29] more extensive and more precise than that of Fisher, also gave null effects, in iron and Permalloy dust, when Fisher's rotation hypothesis formula, or my effect, would have given deflections of 3.5 and $5m$, respectively.

Other experiments related to these, begun some time ago, are again in progress in Pasadena.

While it would carry us too far to enter here into any discussion of experiments on nuclear resonance in magnetic fields, certain experiments analogous to them and closely related to the gyromagnetic experiments discussed above should be mentioned, namely, the very recent ***resonance experiments on the magnetic elements in ferromagnetic bodies*** by Griffiths[30] in England, Snoek[31] in Holland, and Yager and Bozorth[32] in America. These new investigations have not yet become precise, but they nevertheless yield approximate values for the gyromagnetic ratios of the ferromagnetic substances studied, in as good agreement as could be expected with those determined by the more precise methods already described.

IV. COSMICAL ROTATION AND MAGNETIZATION

I shall now return, but very briefly, to cosmical phenomena. Shortly after Rowland's discovery of the magnetic effect of electric convection, John Perry sought to explain terrestrial magnetism by the rotation of the earth's negative charge, implicitly making the assumption that an observer rotating with the earth and an observer fixed in space would see the same *magnetic* field. Rowland pointed out, however, that a charge nearly 10^8 times that actually existing would be needed. Later, Schuster[33] showed that while a fixed observer would see a magnetic field of the earth's type, an observer rotating with the earth, if he saw the right horizontal magnetic intensity, would see a ***vertical intensity with the sign reversed.*** And still later, when the charge of the Kennelly-Heaviside layer, equal and opposite to that of the earth's surface, was taken into account, it became clear that an observer located on the earth would detect no magnetic field, whereas a fixed observer would detect a horizontal magnetic intensity, but no vertical intensity.

To get rid of this relativity difficulty, Sutherland[34] considered the case of a rotating sphere with a positive charge uniformly distributed throughout its volume, and an equal negative charge uniformly distributed over its surface. Outside the surface there would then be no electric field, while the magnetic field, which would therefore appear identical to fixed and moving observers, would have the characteristics of the earth's actual field as we see it. The difficulty is to get the necessary distribution of electric charge.

Centrifugal displacement of negative electricity doubtless occurs, as we have already seen; but calculation indicates that its effect would be billions of billions of times too small. Thermionic displacement of electrons toward the exterior would give an effect nearly as small, and the same is true of gravitational displacement.

In 1924 Angenheister[35] worked out the detailed theory, which is very simple, for Sutherland's charged sphere. Calculating the electric volume density and corresponding surface density necessary to give the earth its known polar magnetic intensity, and making corresponding calculations for the sun, he found the ratio of the electric density to the mass density *identical* for the two. This means that the ratio of angular momentum to magnetic moment, or the gyromagnetic ratio, is the same for both bodies. As Angenheister points out, however, there is no known way in which the necessary electric densities could be produced; and there is no known way in which they could be maintained, as the internal electric field would be extremely intense, about 10^8 v/cm close to the surface.

For the star 78 Virginis and some others H. W. Babcock[4] now finds practically the same gyromagnetic ratio as Angenheister's value for the earth and sun, but the uncertainties are very great. And for one star he finds the magnetic moment to change with time, in ***both magnitude and sign.***[36]

In 1923 H. A. Wilson,[37] following a suggestion

[29] S. J. Barnett, *Proc. Am. Acad. Arts Sci.* **68**, 229 (1933).
[30] J. H. E. Griffiths, *Nature* **158**, 670 (1946).
[31] J. L. Snoek, *Nature* **160**, 90 (1947).
[32] Yager and Bozorth, *Physical Rev.* **72**, 80 (1947).
[33] A. Schuster, *Proc. Physical Soc.* (*London*) **24**, 121 (1912).
[34] W. Sutherland, *Terrestrial magnetism*, vol. V, pp. 73, 1900 ff.
[35] G. Angenheister, *Gött. Nachr.* (1924), p. 220.
[36] H. W. Babcock, *Pubs. Astron. Soc. Pacific* (Oct. 1947).
[37] H. A. Wilson, *Proc. Roy. Soc.* (*London*) (A) **104**, 451 (1923).

of Schuster[38] that all large rotating bodies might be magnetized, assumed that every gravitational unit of mass—$\sim$4 kg—no matter what the material, has in some way an electrostatic unit of charge. On the implicit assumption that there is no screening, and without considering the relativity effect of Schuster (in the case of the earth), Wilson finds, for entirely uniform conditions inside the sphere, a magnetic field intensity at the equator given by

$$H_E = \frac{4}{15}\rho\omega R^2 \text{ emu},$$

where ρ is the electric volume density, ω the angular velocity, and R the radius. This is equivalent, as shown by Blackett,[39] to

$$H_E = G^{\frac{1}{2}}\omega M/5cR \text{ emu},$$

where G is the gravitation constant, and M the mass. This formula gives for both earth and sun a value of H_E about three times too large. It also gives the same gyromagnetic ratio as Angenheister's formula.

If we designate by Q the total charge of the earth, the electric field intensity at the surface (on the assumptions referred to) is $E = Q/R^2$. Wilson sought, in effect, to measure this intensity by a magnetic method, but found no intensity as great as 1/5000 of that calculated. The experiments, however, were made inside the laboratory, whose walls were conductors, and therefore screening should have reduced to zero whatever intensity existed outside.

The application of Wilson's or Blackett's formula to Swann's experiment would give only 10^{-5} times the smallest intensity that Swann could detect (see reference 39).

Another theory has been developed by Elsasser,[40] according to which the magnetization of the earth is produced by thermoelectric currents in the core, given the proper distribution around the rotation axis by Coriolis forces. But the effects are difficult to calculate and far from certain. Cowling[41] has applied this theory to the sun. While he finds an effect in the right direction, it is 10^7 times too small.

It is possible that a variety of effects such as we have considered is primarily responsible, giving magnetization of the right sign but far too small, and that their effects are multiplied, as suggested long ago by Larmor,[42] by a kind of *dynamo action*. But there is no proof of this.

The failure of any known effect to give a satisfactory explanation has led Schuster[43] and others to suggest that perhaps the laws of electrodynamics as commonly accepted must be slightly altered, so as to give considerable effects in large bodies, while the alteration produces no appreciable effect in the case of small ones. But all such theories are purely *ad hoc* and completely speculative.

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[38] A. Schuster, *Brit. Assoc. Advancement Sci. Rept.* (1891).
[39] P. M. S. Blackett, *Nature* **159**, 658 (1947).
[40] W. M. Elsasser, *Physical Rev.* **55**, 489 (1939).
[41] T. G. Cowling, *M. N.* **105**, 166 (1945).
[42] J. Larmor, *Brit. Assoc. Advancement Sci. Rept.* (1919), p. 159.
[43] See also W. F. G. Swann, *Phil. Mag.* **3**, 1088 (1927); T. Schlomka, *Geophysik* **9**, 99 (1933).

It seems to me important that new conceptions and results that are generally accepted by scientific men should be incorporated at once in elementary teaching, provided that they simplify current explanations or assist the formation of a clearer mental picture of natural processes. . . . While no doubt it might be possible and even instructive to approach the study of mechanics by an elaborate series of experiments with pendulums and other moving bodies before presenting Newton's laws of motion, in practice progress is much more definite and rapid if these laws are assumed and are shown at a later stage to be in accord with experience. A similar situation confronts us today with reference to the use of the atomic theory. We are quite certain that the existence of the atom is no longer an hypothesis but a verified fact, and we are confident that the masses of the individual atoms and the number of atoms in a known weight of material are known with errors which are certainly less than 1 percent. In teaching both physics and chemistry, there seems to be no objection to presenting atoms at an early stage as real constituents of matter about whose concrete existence there is no longer room to doubt.—LORD RUTHERFORD, *Sch. Sci. Rev.* **4**, 107 (1923).

The Heavy Radioactive Nuclear Species*

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The genetic relationships among all of the recently discovered heavy radioactive nuclear species[1] and the members of the three naturally occurring radioactive families do not seem to have been pointed out. Similarly, there seems to be no tabulation showing completely all the relationships involved in the recently announced $4n+1$ disintegration series.[2,3] Since the radioactivity of heavy, naturally occurring nuclear species is discussed in most introductory physics textbooks, it seems important to point out the relationships referred to and further to emphasize a point of view which, although by no means new, does not seem to be widely adopted at the introductory physics level.

Figures 1 to 4 show, respectively, all members of the thorium $4n$, neptunium $4n+1$, uranium-radium $4n+2$ and actinium $4n+3$ disintegration series that have been reported in the literature. Isotopes lie along the 60° lines, with Z values as indicated. Note the occurrence of collateral members[2] and the long alpha-sequences in the $4n$ and the $4n+2$ series. If occurrence in nature be ignored, the term "initial member" would seem to have little if any significance. Clearly, future additions to Figs. 1 to 4 may be expected. In each figure the double ring indicates the longest-lived member of the series. A question mark indicates that the particular disintegration has been inferred but not observed. A heavy ring indicates a stable chain product.

Table I shows the decay schemes, periods, and literature references for the radioactive species shown in Figs. 1 to 4. The information in the figures and the table permits one to emphasize, in teaching, that there is no fundamental difference, from the point of view of nuclear structure, between "natural" and "artificial" radioactivity. The artificially prepared, alpha-emitting, Pu^{238} is not essentially different[3a] from the naturally occurring Ra^{226}. Similarly, the naturally occurring beta-emitter Th^{234}(UX1) is not different in character from the cyclotron- or pile-produced beta-emitters Np^{237} or P^{32}. The occurrence of K-capture decay processes should be noted.

The existence in nature of any of the species listed in Table I depends on the "fortuitous" fact that only three of them—U^{238}, U^{235} and Th^{232}—have half-lives that are sufficiently long for them to have survived a period when (we assume) there

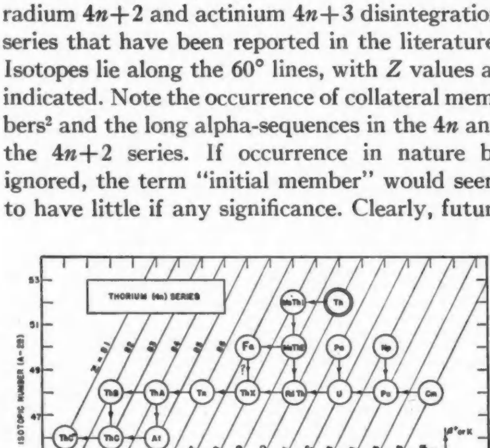

Fig. 1. Reported members of the thorium $4n$ disintegration series.

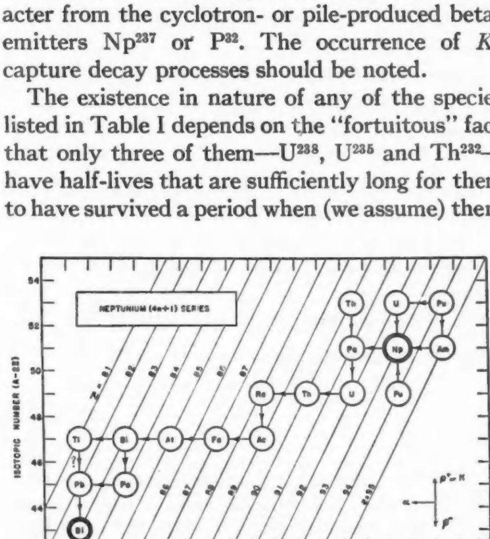

Fig. 2. Reported members of the neptunium $4n+1$ disintegration series.

* Contribution (10p-47) from the Department of Physics.

[1] Isotopes of elements 82, 83, 85, 87, 90, 91, 92, 93, 94, 95 and 96 have been discovered during the past decade.

[2] E. Hagemann, L. I. Katzin, M. H. Studier, A. Ghiorso and G. T. Seaborg, *Physical Rev.* **72**, 252 (1947); all members of this series have mass numbers of the form $4n+1$.

[3] A. C. English, T. E. Cranshaw, P. Demers, J. A. Harvey, E. P. Hinks, J. V. Jelley and A. N. May, *Physical Rev.* **72**, 253 (1947).

[3a] On the basis of such data as have been published, some, but not all, of the recently discovered species fail to fit on Geiger-Nuttall curves. G. T. Seaborg (private communication) states that a "main line of decay" seems to exist, characterized by the fact that its alpha-emitting members are correctly described by the Geiger-Nuttall relationship.

TABLE I. Heavy, chain-disintegrating nuclear species.

Element	Mass number A	Decay	Period	Reference
Thorium 4*n* Series				
Cm	240	α	30 d	5
Np	236	β^-	20 hr	5
Pu	236	α	?	6
Th	232	α	1.39×10^{10} yr	4
Pa	232	β^-, γ	1.6 d	6
U	232	α	30 yr	6
Ra(MsTh1)	228	β^-	6.7 yr	4
Ac(MsTh2)	228	β^-, α, γ	6.13 hr	4
Th(RdTh)	228	α, γ	1.90 yr	4
Fa	224	$(\beta^- ?)$	?	4
Ra(ThX)	224	α	3.64 d	4
Rn(Tn)	220	α	54.5 sec	4
Po(ThA)	216	α, β^- (0.014%)	0.158 sec	4
At	216	α	<54 sec	4
Pb(ThB)	212	β^-, γ	10.6 hr	4
Bi(ThC)	212	β^- (66.3%), α, γ	60.5 min	4
Po(ThC′)	212	α	3×10^{-7} sec	4
Tl(ThC″)	208	β^-, γ	3.1 min	4
Po	208	α	~3 yr	8*
Tl	204	β^-	3.5 yr	8*
Bi	204	K, γ	12 hr	9*
Pb	204	IT, γ	68 min	9*
Neptunium 4*n*+1 Series				
Pu	241	α, β^-	"long"	6, 7
Am	241	α	500 yr	5
U	237	β^-, γ	6.8 d	5
Np	237	α	2.25×10^{6} yr	5
Pu	237	K	?	6
Th	233	β^-	23.5 min	4
Pa	233	β^-, γ	27.4 d	4
U	233	α	1.63×10^{5} yr	2
Th	229	α	7×10^{3} yr	2, 3
Ra	225	β^-	14.8 d	2, 3
Ac	225	α	10.0 d	2, 3
Fa	221	α	4.8 min	2, 3
At	217	α	0.018 sec	2
Bi	213	β^- (96%), α	47 min	2
Po	213	α	3.2×10^{-6} sec	3
Tl	209	$(\beta^- ?)$	?	2, 3
Pb	209	β^-	3.3 hr	2, 3, 9

* These nuclei are not shown in Fig. 1. A genetic relationship to the main sequence has not yet been established. Also, At^{215} [Karlik and Bernert, *Naturwiss.* 32, 44 (1944)] is not shown in the figures or the table.

TABLE I.—*Continued.*

Element	Mass number A	Decay	Period	Reference
Uranium-Radium 4*n*+2 Series				
Am	242	β^-	18 hr	7
Cm	242	α	5 mo	5
U(UI)	238	α	4.51×10^{9} yr	4
Np	238	β^-	2.0 d	5
Pu	238	α	50 yr	5
Th(UX1)	234	β^-, γ	24.5 d	4
Pa(UX2)	234	β^-, γ	1.14 min	4
Pa(UZ)	234	β^-, γ	6.7 hr	4
U(UII)	234	α	2.69×10^{5} yr	4
Np	234	K, γ	4.4 d	5
Th(Io)	230	α, γ	8.3×10^{4} yr	4
Ra	226	α, γ	1590 yr	4
Rn	222	α	3.825 d	4
Po(RaA)	218	β^- (0.04%), α	3.05 min	4
At	218	α	several sec	4
Pb(RaB)	214	β^-, γ	26.8 min	4
Bi(RaC)	214	β^-, α (0.04%)	19.7 min	4
Po(RaC′)	214	α	1.5×10^{-4} sec	4
Tl(RaC″)	210	β^-	1.32 min	4
Pb(RaD)	210	β^-, γ	22 yr	4
Bi(RaE)	210	β^-	5.0 d	9
Po	210	α, γ	140 d	4
Tl	206	β^-	4.23 min	8
Bi	206	K, γ	6.4 d	8
Po	206	K (90%), α, γ	9 d	8
Actinium 4*n*+3 Series				
U	239	β^-, γ	23.5 min	6
Np	239	β^-, γ	2.33 d	6
Pu	239	α	2.4×10^{4} yr	5
U(AcU)	235	α	7.07×10^{8} yr	4
Np	235	K	8 mo	5
Th(UY)	231	β^-	24.6 hr	4
Pa	231	α, γ	3.2×10^{4} yr	4
Ac	227	α (1%), β^-	13.5 yr	4
Th(RdAc)	227	α, γ	18.9 d	4
Fa(AcK)	223	β^-, γ	21 min	4
Ra(AcX)	223	α, γ	11.2 d	4
Rn(An)	219	α	3.92 sec	4
Po(AcA)	215	α	1.83×10^{-3} sec	4
Pb(AcB)	211	β^-, γ	36.1 min	4
Bi(AcC)	211	β^- (0.32%), α	2.16 min	4
Po(AcC′)	211	α	5×10^{-3} sec	4
At	211	K (40%), α	7.5 hr	4
Tl(AcC″)	207	β^-, γ	4.76 min	4
Po	207	α (0.01%), K, γ	5.7 hr	8
Pb	203	K, γ	52 hr	9

was a general distribution of many radioactive species. The occurrence of Pu^{239} in nature (1 part by mass in 10^{14} parts of pitchblende) has been ascribed[7] to its continual production by spontaneous fission of U^{238}. It, too, would not occur in nature if the half-life of U^{238} were much shorter. The occurrence of the very long-lived species $K^{40}(7\times10^{8}$ yr), $Rb^{87}(6\times10^{10}$ yr), $Sm^{148}(1.5\times10^{11}$ yr) and $Lu^{176}(7.3\times10^{10}$ yr) should also be pointed out as part of the general picture.

A further advantage of presenting information of the sort shown in Figs. 1 to 4 when radioactivity is discussed with beginning students is that it links smoothly what is very old in nuclear physics with what is very new, and forms a con-

[4] G. T. Seaborg, *Rev. Mod. Physics* **16**, 1 (1944).
[5] G. T. Seaborg, *Science* **104**, 379 (1946).
[6] J. M. Cork, *Radioactivity and nuclear physics* (Van Nostrand, 1947).
[7] G. T. Seaborg, *Chem. Eng. News* **25**, 358 (1947).
[8] D. H. Templeton, J. J. Howland and I. Perlman, *Physical Rev.* **72**, 758 (1947).
[9] D. H. Templeton, J. J. Howland and I. Perlman, *Physical Rev.* **72**, 766 (1947).

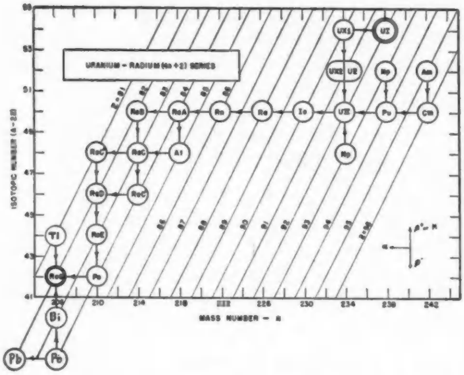

FIG. 3. Reported members of the uranium-radium $4n+2$ disintegration series.

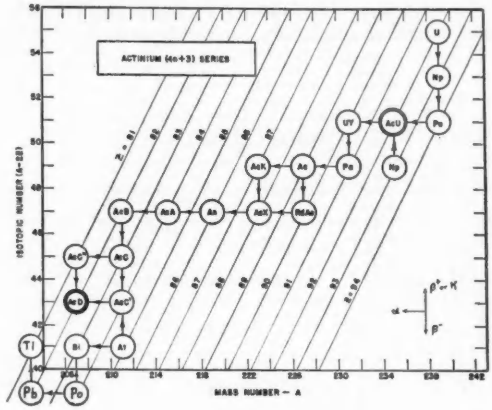

FIG. 4. Reported members of the actinium $4n+3$ disintegration series.

venient point of departure for the discussion of artificial transmutation, including fission. It provides an easy and logical path whereby students may be shown a glimpse of the research frontiers, a desirable situation if one is to get across the concept of "what physics is really like."

The author wishes to thank Dr. O. H. Blackwood for criticizing the manuscript. Dr. G. T. Seaborg was kind enough to answer several questions that came up and to look over the manuscript; he did not, however, check the figures and the table in detail.

Rigid Body Sings

'Gin a body meet a body
Flyin' through the air,
'Gin a body hit a body,
Will it fly?—and where?
Ilka impact has its measure,
Ne'er a ane hae I,
Yet a' the lads they measure me,
Or, at least, they try!

'Gin a body meet a body
Altogether free,
How they travel afterwards
We do not always see:
Ilka problem has its method
By analytics high;
For me, I ken na ane of them,
But what the waur am I?

JAMES CLERK MAXWELL.

A Precise Laboratory Exercise Using a Vacuum Tube Bridge

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IT is one of the legitimate objects of a laboratory exercise to obtain data in substantially precise agreement with the values to be expected from theoretical considerations. For several reasons, this desirable attribute is commonly absent from exercises involving vacuum tubes. A vacuum tube is not inherently stable, and its characteristics can be altered by momentary overloads such as may occur when a student has made some error in circuit connections in the course of a lengthy exercise. The exercise should be designed, therefore, to permit ready remeasurement of the appropriate characteristics. Again, the customary "linear" analysis of vacuum tube circuits is based upon the assumption of *infinitesimal* changes in the electrode potentials. When the data are to be obtained from changes of meter readings, these changes must, in practice, be large enough to make the linear analysis more or less inapplicable, and to rule out the possibility of precise comparison with theoretical values.

The last-mentioned, and most fundamental, difficulty, is avoided by the use of a vacuum tube bridge. However, this well-known device is designed to measure only the amplification factor μ, the plate resistance r_p and the transconductance g_m, using small signal voltages. This article describes the extended application of the vacuum tube bridge to the measurement of the over-all properties of some of the fundamental amplifying circuits.

The bridge manufactured by the General Radio Company (Type 561) has several distinct advantages from the pedagogic point of view.[1] The values of μ, r_p and g_m are independently determined; hence the relation

$$\mu = r_p g_m$$

may be verified. The other advantages may be lumped under the heading of adaptability. With the addition of an easily constructed adapter for this bridge, one may measure with precision the following quantities:

(i) μ, r_p and g_m;
(ii) The voltage gain A and the output resistance R_0 of a resistance-loaded triode amplifier;
(iii) The effective voltage gain A' and the effective output resistance R_0' of an amplifier with negative current feedback;
(iv) A and R_0 of a cathode follower;
(v) A' and R_0' of an amplifier with negative voltage feedback;
(vi) μ, r_p and g_m of a two-tube arrangement for which g_m and μ are negative.

In addition, using this same arrangement, one may determine

(vii) the criterion for oscillation of a negative-resistance oscillator.

The circuits used are such that an expression for each of the aforementioned quantities can be obtained by linear analysis. In each instance, the experimental values are found to agree with the theoretical values within 2 percent.

As constructed for use with this department's Type 561-D bridge, the adapter (Fig. 1) consists of an octal socket mounted upon a Bakelite slab. The lead from each of the socket connections is interrupted to permit the insertion of resistors such as the General Radio Type 500, with banana plug connectors, or of shorting bars, as the circumstances require. This adapter replaces the octal socket adapter furnished by the manufacturer and, like it, makes connection to the bridge proper by means of eight sub-panel banana plugs. It should not be difficult to construct a similar adapter for the earlier models of the Type 561 bridge.

The simplified diagram, Fig. 2(a), illustrates the measurement of amplification factor. Two small signal voltages, e_1 and e_2, differing in phase by 180°, are inserted into the grid and plate circuits, respectively. Their magnitudes are then adjusted by means of calibrated attenuators to produce a null of voltage across the tuned output transformer T. When this is the case, the plate current variation is zero, and, in accordance with

[1] H. J. Reich, *Theory and applications of electron tubes* (McGraw-Hill, 1939). The basic principle of operation of the bridge is discussed adequately in secs. 15–32.

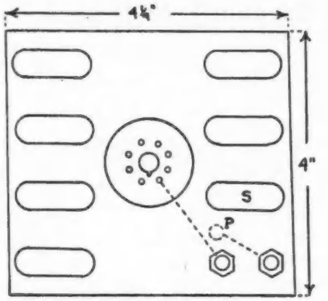

FIG. 1. Plug-in adapter for G. R. Type 561-D bridge. One of the short circuit bars, S, has been removed to show the connection to socket pin *1* and to the corresponding sub-panel plug P, which makes connection to the bridge proper.

its definition, the amplification factor is given by the ratio of e_2 to e_1, and can be read directly from the settings of the attenuator knobs.

The modification permitted by the adapter is shown in Fig. 2(*b*). The balance condition is now that e_2 shall be equal to e_0, the output voltage of the amplifier. The voltage gain e_0/e_1 is therefore numerically identical with e_2/e_1, and can be read directly in the same manner as before.

In its unmodified form, the bridge measures r_p by balancing the voltage i_pr against a signal e_1, to produce zero output from the transformer (Fig. 3). The magnitude of r and the adjustments of e_1 and e_2 are such that the attenuator settings indicate r_p directly. When modified by the insertion of the load resistor R_L (Fig. 2(*b*)), the bridge indicates the effective resistance of r_p and R_L in parallel, which is, of course, the output resistance of the one-stage amplifier.

When the bridge is adjusted to indicate transconductance, Fig. 3 applies again, except that e_1 appears in series with the grid circuit.

A switch on the bridge panel reverses the phase of e_2 with respect to e_1. This makes possible the measurement of negative values of the characteristics of the vacuum tube and of the amplifier.

A source of adjustable plate and grid voltages, a 1000-c/sec tone source, and a bridge balance detector are needed, in addition to the bridge itself and the circuit elements mentioned below. The tubes under test are conveniently the 6J5 and its double-triode counterpart, the 6SN7GT.

The procedure, in outline, follows.

(1) Following the manufacturer's instructions, the student measures μ, r_p and g_m, and verifies their theoretical relation. This measurement is simply performed, and is repeated whenever there is occasion to suppose the values to have changed.

(2) A 10,000-ohm resistor R_L is plugged into the plate lead, and a blocking condenser is added as in Fig. 2(*b*). The connections to the output transformer will have been made correctly if the student is instructed to follow the manufacturer's directions for the measurement of the resistance between the free end of the condenser and ground. Allowance must be made for the voltage across R_L, and the plate supply voltage should be raised accordingly. With these connections, adjustment of the bridge as if to measure amplification factor will yield, instead, the magnitude of the voltage gain A, and adjustment of the bridge as if to measure plate resistance will yield, instead, the magnitude of the output resistance R_0. These values are to be compared with those obtained from the equations

$$A=\mu R_L/r_p+R_L$$

and

$$1/R_0=(1/r_p)+(1/R_L),$$

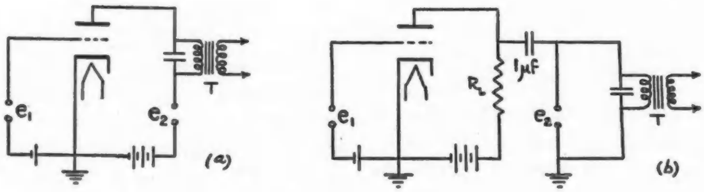

FIG. 2. (*a*) Simplified diagram to illustrate the method of measurement of amplification factor; (*b*) modification to permit measurement of gain. The condenser, T, and e_2 should be represented as being in series, and not as shown.

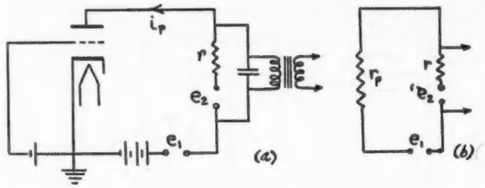

FIG. 3. (a) Simplified diagram to illustrate the method of measurement of plate resistance; (b) equivalent circuit.

derived by the customary linear analysis of this circuit.[2]

(3) The addition of a 500-ohm cathode resistor R_k converts the arrangement of Fig. 2(b) into a current feed-back amplifier. Both grid and plate voltages must now be adjusted to allow for the drop across R_k. The measured gain A' and output resistance R_0' are now to be compared with the values obtained from the equations

$$A' = A/(1 - A\beta)$$

and

$$\frac{1}{R_0'} = \frac{1}{R_L} + \frac{1}{r_p + (\mu+1)R_k},$$

where $\beta = -R_k/R_L$, and where the value of A is that determined in procedure (2).

The change in gain and in output resistance because of the addition of R_k are consistent with the following statement: the vacuum tube behaves as if its amplification factor has not altered, but its plate resistance has increased by the amount $(\mu+1)R_k$. A direct check of the correctness of this statement may be made by returning to procedure (1) and measuring the apparent amplification factor and plate resistance when R_k is included in the circuit.

(4) By removing R_L, and transferring the output connection to R_k by way of a 100-μf blocking condenser, the student converts the amplifier into a cathode follower. The gain A is now found to be negative, because the input and output voltages are now in phase, instead of being 180° out of phase as before. The appropriate analytic expressions are

$$A = \frac{\mu R_k}{r_p + (\mu+1)R_k}$$

and

$$\frac{1}{R_0} = \frac{1}{R_k} + \frac{\mu+1}{r_p}.$$

(5) Figure 4 shows a voltage feed-back amplifier of the parallel feed-back type. The gain A (without feedback) is measured with the feed-back resistor R_f connected to ground, to allow for the fact that the feed-back network acts as an additional plate load paralleling R_L. The error resulting from omitting R_g from this correction is negligible. Then the gain is measured with R_f connected to the grid. This new value of the gain must be corrected, because the measuring signal is inserted into a grid circuit consisting of R_g, R_f and R_L in series, and only the portion of this signal that is across $R_f + R_L$ is applied between grid and ground. The correction, therefore, consists in multiplying the amplification factor reading of the bridge by $(R_f + R_g + R_L)/(R_f + R_L)$. This corrected value of A' and the measured value of R_0' are to be compared with

$$A' = A/(1 - A\beta)$$

and

$$\frac{1}{R_0'} = \frac{1}{R_L} + \frac{1-\mu\beta}{r_p} + \frac{1}{R_g + R_f},$$

where $\beta = -R_g/(R_g + R_f)$. This expression for R_0' is the one usually found in the literature, except that the contribution of $R_f + R_g$, not being negligible, is accounted for by the last term.

(6) The transconductance of the arrangement of Fig. 5 may be defined as $g_m \equiv i_{p2}/e_1$. That this may be expected to be negative may be demonstrated qualitatively as follows. A *rise* in potential of the first grid causes a rise in potential of the cathodes. Since the potential of the second

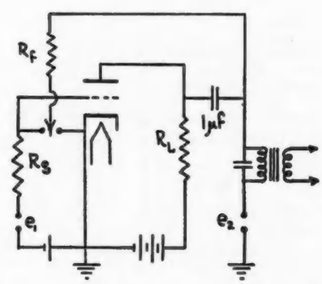

FIG. 4. Voltage feed-back amplifier.

[2] J. Millman and S. Seely, *Electronics* (McGraw-Hill, 1941), ch. XVII.

grid cannot vary relative to ground, its potential relative to its own cathode is decreased, with a consequent *decrease* of the plate current of the second tube.

The symbols e_g, e_p and i_p in Fig. 5 refer to the alternating components of the voltages and plate currents of the two triodes. Suppose the e_2-terminals to be short-circuited. The linear approximation to the properties of the tubes yields the equations

$$i_{p1}=g_{m1}e_{g1}+(e_{p1}/r_{p1})$$

and

$$i_{p2}=g_{m2}e_{g2}+(e_{p2}/r_{p2}).$$

The network equations, assuming the instantaneous polarities and directions of current to be as shown, are

$$e_{p1}=e_{p2}=e_{g2}=-(i_{p1}+i_{p2})R_k$$

and

$$e_s=e_{g1}+(i_{p1}+i_{p2})R_k.$$

Combining these equations to obtain the ratio i_{p2}/e_1, we have, in accordance with the foregoing definition,

$$g_m=-\frac{R_kg_{m1}[g_{m2}+(1/r_{p2})]}{1+R_k[g_{m1}+g_{m2}+(1/r_{p1})+(1/r_{p2})]}.$$

An expression for the over-all output resistance i_{p2}/e_2 (evaluated with e_1 equal to zero) could be similarly obtained, as could an expression for the over-all amplification factor e_2/e_1 (evaluated with i_{p2} equal to zero). However, these expressions would not be especially instructive or useful.

This two-tube arrangement offers a particularly striking verification of the interrelation of the constants μ, r_p and g_m, because the measured value of g_m will be found to be negative, that of r_p will be found to be positive, and that of μ turns out, as it should, to be negative.

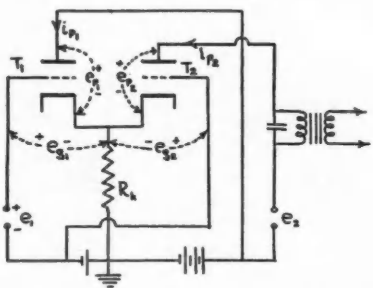

FIG. 5. An example of negative transconductance.

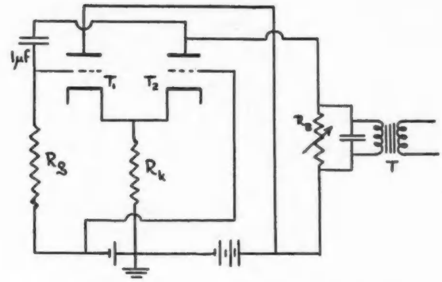

FIG. 6. Negative resistance oscillator.

As an example of the numerical agreement that may be expected, in a particular instance, with $R_k=500$ ohms and a total cathode current of 15 ma, the measured value of g_m was -830 µmhos. The value computed from the formula given above was -836 µmhos, a difference of less than 1 percent.

(7) The addition of a coupling condenser and a grid resistor (Fig. 6) makes the output resistance negative. Qualitatively, again, a signal inserted into the plate circuit of tube T_2 in such a sense as to raise the plate potential will, because of the coupling condenser, raise the potential of the grid of the tube T_1 also. The consequent rise of cathode potential will, as before, cause a *reduction* of plate current in T_2 that will overwhelm the effect of raising the plate potential of that tube. The ratio of potential change to current change is therefore negative.

The condition for self-sustained oscillation is that the resistance of the parallel combination of the negative resistance and the load resistance shall be zero or negative. The (resistive) impedance of the tuned transformer at its resonant frequency of 1000 c/sec was independently measured as 34,500 ohms, while the negative resistance is only about 1400 ohms. Consequently, sustained oscillations will be obtained immediately. In accordance with the manufacturer's instructions, a resistance, placed across the tuned circuit, is adjusted to a low enough value to stop the oscillations, and the magnitude of the negative resistance is determined in the usual fashion.

To obtain an expression for the magnitude of the output resistance, we may once more write the equations that represent the linear approxi-

mation to the properties of the triodes, and, in addition, the following network equations:

$$e_{g1}=e_{p2}=e_2-(i_{p1}+i_{p2})R_k$$

and

$$e_{g2}=e_{p1}=-(i_{p1}+i_{p2})R_k.$$

The transformer impedance does not appear in these equations, because there is no voltage across it when the bridge is balanced. In addition, the algebra is simplified if R_g is temporarily assumed to be infinite. Combining these equations to obtain the ratio i_{p2}/e_2, which is by definition the over-all output resistance, and adding a term to represent the conductance of R_g, we have

$$\frac{1}{R_0}=\frac{-R_k[g_{m1}g_{m2}-(1/r_{p1}r_{p2})]}{1+R_k[g_{m1}+g_{m2}+1/r_{p1}+(1/r_{p2})]}+\frac{1}{R_g}.$$

The measured value of the negative resistance is to be compared with that obtained from substitution into this expression.

The resistance may now be adjusted to the value that just barely stops the oscillations. The student may then verify the relation

$$\frac{1}{R_0}+\frac{1}{R_T}+\frac{1}{R_B}=0,$$

where R_0, R_T and R_B are the negative resistance, the resistance of the tuned transformer, and the adjustable resistance, respectively. As a further illustration of the precision to be expected, the respective quantities, expressed as conductances, were, in a particular case, $G_0=-719$ μmhos, $G_T=29$ μmhos, and $G_B=694$ μmhos; the negative conductance is thus seen to be very nearly equal to the total positive conductance.

The author has made the exercise as described herein a part of an advanced laboratory course during the past year. The results obtained by the students have been uniformly and gratifyingly precise.

Demonstrating the Diamagnetism and Paramagnetism of Liquids

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THE force on a ferromagnetic material in a magnetic field is large and easy to demonstrate. It is a much more difficult matter to demonstrate diamagnetism or paramagnetism, particularly of ordinary liquids. The force with which even a strong permanent magnet acts on them is so minute that the torsion balance usually employed to disclose it must be very delicate. Balances of the necessary sensitivity are neither conveniently portable nor rugged enough for lecture demonstration. Furthermore, the deflections, even after optical magnification, are too small to give a convincing demonstration. As far as lecture-table demonstration is concerned, emphasis is usually placed on ferromagnetic phenomena, so that students are likely to gain the erroneous impression that ferromagnetic solids are the only substances exhibiting magnetic properties.

The device to be described can be used only with liquids; however, this is not a serious handicap for the purposes of a striking demonstration, because the common liquids are least likely to be associated with magnetic effects in the minds of students. In principle the device is related to the well-known methods of Gouy and Quincke for measuring magnetic susceptibilities.

1. Methods of Gouy and Quincke[1]

In the method of Gouy an elongated cylinder of the material whose susceptibility is to be

[1] For details of theory, related methods and references, see: E. C. Stoner, *Magnetism and matter* (Methuen, 1934); P. W. Selwood, *Magnetochemistry* (Interscience Pub., 1943); W. Klemm, *Magnetochemie* (Akademische Verlagsgesellschaft, 1936); S. S. Bhatnagar and K. N. Mathur, *Physical principles and applications of magnetochemistry* (Macmillan, 1935); L. F. Bates, *Modern magnetism* (Cambridge Univ. Press, 1939).

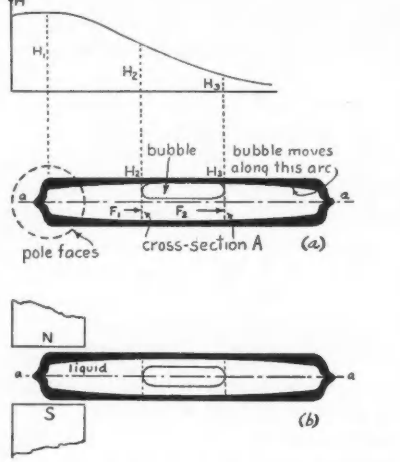


FIG. 1(a). A spirit-level tube containing a diamagnetic or paramagnetic liquid is shown with one end between the poles of a magnet. The magnetic force on a diamagnetic liquid is F_1 at the left-hand end of the bubble and F_2 at the right-hand end, and the bubble moves to the left. In a paramagnetic liquid the bubble moves away from the magnet. A graph of the magnetic field is also shown; (b). Same tube viewed from above. The curvature of the ground interior of the tube is highly exaggerated.

measured is suspended from one arm of a balance, with the lower end of the cylinder between the poles of a magnet. This end of the cylinder is in the uniform field H_1 between the poles, while the upper end, remote from the poles, is in a field H_2. With liquids a cylindrical column is obtained by placing them in a glass tube. If the force on the glass and the effect of the medium—air—in which the cylinder is immersed are neglected, the force in dynes on the cylinder is given by $F=\frac{1}{2}(H_1^2-H_2^2)\kappa A$, where κ is the volume susceptibility of the liquid, and A the cross-sectional area of the cylinder. This force is determined by adding weights to the other arm of the balance. For diamagnetic liquids this weight is of the order of 1 mg for a field of 10^4 gauss between the magnet poles.

In the method of Quincke the liquid is placed in a U-tube, one arm of which is wide and the other a capillary. The latter is placed between the poles of a magnet with the liquid meniscus at the center of the region of uniform field H_1. The liquid in the part of the column remote from the poles is in a field H_2. The force on the liquid column, as in the method of Gouy, is given by $F=\frac{1}{2}(H_1^2-H_2^2)\kappa A$, where A is the cross-sectional area of the liquid in the capillary, the other symbols having the same significance as before. This force causes the surface of the liquid in the capillary to be displaced by a distance Δh, which is a measure of the pressure P developed in the liquid by the magnetic force. If the magnetic force on the gas above the liquid is neglected,

$$P\equiv F/A=\tfrac{1}{2}(H_1^2-H_2^2)\kappa=g\Delta h.$$

For diamagnetic liquids Δh is of the order of 1 mm for a field of 10^4 gauss. This small displacement and the small weights involved in the Gouy method show clearly how small the diamagnetic effects are.

2. Behavior of Spirit-Level in a Magnetic Field

The writer has found that when a spirit level is placed with one end between the poles of a permanent magnet a considerable horizontal displacement of the air bubble occurs. With a field of about 5×10^3 gauss a displacement of 5 cm is obtained with a level of moderate sensitivity. This seems rather astonishing until one calculates that the vertical displacement of the bubble in a tube with a 100-m radius is only 5×10^{-5} cm for a 1-cm horizontal displacement.

Before proceeding to discuss the mechanics involved, it may be worth while to say a few words about spirit levels. While these are still important for the precise leveling of surveying and astronomical instruments and cathetometers, they are otherwise relatively unimportant in modern physical measurements. Consequently, their construction and high sensitivity are not generally known today.

Contrary to common opinion, the spirit-level tube is not made by bending a cylindrical glass tube and thereby making its long axis an arc of large radius. It is actually made by grinding the inside surface of a glass tube to give

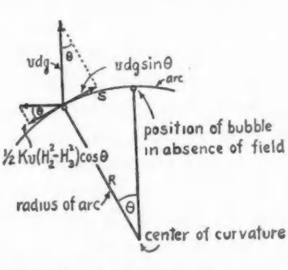


FIG. 2. Diagram showing the equilibrium of the magnetic and gravitational buoyant forces on a bubble. The curvature of the arc along which the bubble moves is highly exaggerated.

it the shape of a highly elongated barrel, as shown in an exaggerated way in Fig. 1(*a*). In a tube ground in this way the bubble always moves along an arc of the same radius, and the center of curvature of the arc is always below the bubble, regardless of how the tube is oriented about its horizontal axis *aa*.

The sensitivity of a level is defined as the angle—measured in seconds of arc—through which the level tube must be tilted in order to make the bubble move 1 division (usually 2 mm) of the scale engraved on the tube. The level tubes used by the writer had sensitivities ranging from 10 to 37 sec, but they can be made with a sensitivity of 1 sec or less.

Figure 1(*a*) shows in elevation a level tube, one end of which is shown located between the poles of a permanent magnet where the field strength is H_1. The bubble is shown in the equilibrium position it occupied just before the magnet was put in position. Consider the end of the bubble nearest to the magnet to lie in a plane drawn perpendicular to the long axis *aa* of the tube. The magnetic field over the cross section of the tube here is taken as uniform over the whole cross section and of strength H_2. Similarly, the other end of the bubble lies in a plane over which the field is H_3. A diagrammatic graph of the magnetic field in relation to the tube is also shown in Fig. 1(*a*). Figure 1(*b*) shows a top view of the same arrangement. If the liquid in the tube is diamagnetic, there is a force acting on it urging it away from the magnet.

Since the arc along which the bubble moves has a radius of 100 m or more, the cross-sectional area A of the liquid column is sensibly uniform everywhere. The magnetic forces over the cross section of the liquid are $F_1=\frac{1}{2}(H_1^2-H_2^2)\kappa A$ at the left-hand end of the bubble and $F_2=\frac{1}{2}(H_1^2-H_3^2)\kappa A$ at the right-hand end. The corresponding pressures P_1 and P_2 are $P_1=\frac{1}{2}(H_1^2-H_2^2)\kappa$ and $P_2=\frac{1}{2}(H_1^2-H_3^2)\kappa$. It will be noted that the pressure increases in the direction away from the magnet; and the pressure difference P_2-P_1 between the two ends of the bubble is $\frac{1}{2}(H_2^2-H_3^2)\kappa$. Thus the liquid develops, so to speak, a "weight" in a horizontal direction away from the magnet, and the bubble will be acted upon by a buoyant force in the opposite direction. This buoyant force is $(P_2-P_1)v=\frac{1}{2}(H_2^2-H_3^2)\kappa v$, where v is the volume of the bubble, and causes the bubble to move toward the magnet. In a paramagnetic liquid the bubble moves away from the magnet because all the forces are reversed.

As the bubble moves toward the magnet it travels along an arc-shaped incline until a position of equilibrium is reached. Here the component along the incline of the magnetic buoyant force is equal to the corresponding component of the gravitational buoyant force. The forces are shown in the diagram of Fig. 2, in which the bubble is shown for convenience as very small. It should be noted that H_2 and H_3 now refer to the magnetic fields at the left- and right-hand ends of the bubble in the new position. From the diagram it may be seen that

$$\tfrac{1}{2}(H_2^2-H_3^2)\kappa v\cos\theta=vdg\sin\theta,$$

where θ is the angular displacement of the bubble from its original position of equilibrium referred to the center of curvature of the arc along which the bubble moves, d is the density of the liquid, and g is the acceleration due to gravity. Solving for the volume susceptibility κ, we get

$$\kappa=2gd\tan\theta/(H_2^2-H_3^2).$$

The angle θ involved is very small, so that $\tan\theta$ may be replaced by θ. Furthermore, if we let the displacement of the bubble along the arc be S and the radius of curvature of the arc be R, then $\theta=S/R$, and we obtain

$$\kappa=2gdS/R(H_2^2-H_3^2).$$

Solving for S, we get

$$S=\kappa R(H_2^2-H_3^2)/2gd.$$

This last expression permits an estimate of the

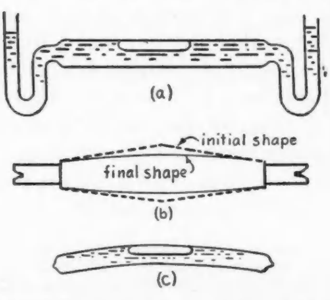


FIG. 3. (*a*) A level tube arranged to permit changing the liquid; (*b*) steel rod for grinding interior of tubes; (*c*) level tube made of curved glass tube; the curvature is exaggerated; such a tube can be used only with the convex side up.

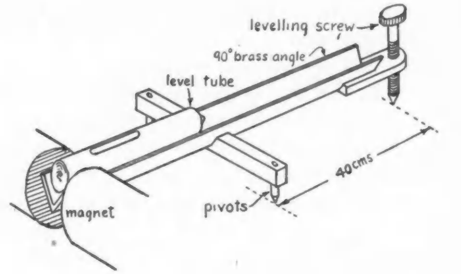


FIG. 4. Leveling arrangement to support level tube. The magnet is shown in position to deflect the bubble.

sensitivity to be made. Take $R=400$ m, $d=1$ g/cm^3, $g=10^3$ cm/sec^2, $\kappa=10^{-6}$, and $H_2^2-H_3^2=2\times10^5$ gauss. This gives for S, the displacement of the bubble, 40 cm.* It may be noted that $R=400$ m does not yet represent the most sensitive level tube (for a 1-sec level, R would be 466 m), and 10^{-6} is the usual order of magnitude of the susceptibility κ of diamagnetic liquids, while a value for $(H_2^2-H_3^2)$ of 2×10^6 gauss can readily be realized. Observations made on level tubes of lower sensitivity (smaller R) confirm this estimate of the sensitivity attainable.

3. Experimental

The experimental work was aimed at devising a method for rapidly testing a liquid to determine whether it was diamagnetic or paramagnetic, without any concern for the actual magnitude of the susceptibility. As the work progressed it became clear that a level tube could be used as a simple means of demonstrating diamagnetism or paramagnetism, and as a means of comparing quantitatively the susceptibility of one liquid with that of another used as a standard. However, the latter application requires further investigation, and this report will concern itself with the use of level tubes for exhibiting diamagnetism or paramagnetism for pedagogic purposes.

The magnets were strong Alnico permanent magnets formerly used to provide the magnetic field for ultra high-frequency magnetrons. One of them had a gap of 1.8 cm and a field in the gap of about 7000 gauss. The other, with a gap of 3.3 cm, had a field of 2500 gauss.

The level tubes were constructed of 14-mm Pyrex tubing, as shown in Fig. 3(*a*). The **U**-tubes shown were sealed on after the tube was properly ground and served as a means of changing the liquid and the size of the air bubble. The inside of the tube was given the elongated barrel shape by lapping with carborundum powder and water on a steel rod rotating in a lathe. The active part of the steel rod was 5 in. long, with an average diameter about 1 mm smaller than the inside diameter of the glass tube. It was given a suitable contour by first turning it in the form of a double truncated cone, as shown in Fig. 3(*b*). The thicker portion at the center of the rod is only a few thousandths of an inch thicker than the ends. Then it is filed and polished while rotating in the lathe until it has the proper shape. The glass tube is constantly rotated during the grinding process, keeping the inner wall constantly in contact with the lapping tool.

Whether levels are ground in this way commercially the writer was unable to learn. The making of level tubes seems to be a rather secret art, for no account of the process could be found in any literature accessible to the writer. However, a number of tubes satisfactory for the purpose in view were made as described. They had a

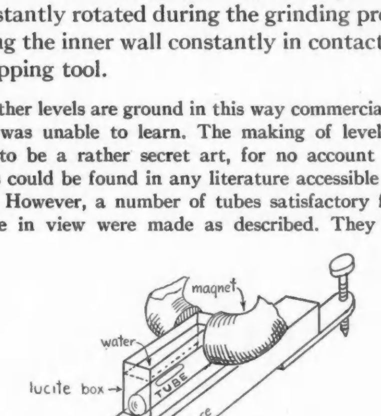


FIG. 5. The upper diagram shows the level tube in a Lucite box filled with water for projection. The lower diagram shows the same set-up and the optical arrangement for projection.

* A displacement of this magnitude cannot, of course, be realized in an actual level tube which is too short to permit such deflections. The bubble moves a distance of several cm and assumes a new equilibrium position in which most of volume of the bubble will be within the pole-gap of the magnet. All the calculations here are semi-quantitative. For precision measurements of susceptibility by this method, it is necessary to take into account the magnetic force on the gas bubble itself.

sensitivity ranging from 10 to 37 sec. If one is willing to sacrifice some convenience of use, the grinding may be eliminated. Almost any length of glass tubing picked at random will have enough curvature to serve as a level tube, but it can be used in one position only, namely, that for which the convex side of the long axis faces up, as shown in Fig. 3(*c*). There may also be slight defects at the inner surface and these may cause the bubble to stick in places.

The level tube is filled with alcohol or ether, with an air bubble 2 to 3 cm long remaining. The tube is placed for convenience of leveling on a piece of brass angle (Fig. 4) sufficiently narrow to permit the pole gap of the magnet to pass around it. The brass angle is provided with a pair of pointed legs to serve as pivots at one end, and a leveling screw is provided at the other end by means of which the level tube can be tilted to bring the bubble near the center of the tube.

If now a permanent magnet is brought up, embracing the tube but not touching it, several centimeters from the bubble, the latter will move through a distance of several centimeters, depending upon the sensitivity of the level tube. It is possible by placing the bubble within the gap apparently to "pull" the bubble from one end of the tube to the other. One gets from this the impression that it is the bubble that is being acted upon by the field, and this seems plausible in view of the paramagnetism of air. However, the air may be replaced by hydrogen or nitrogen without any observable change. Actually, of course, the mass of the oxygen in the bubble is small compared with the mass of the surrounding liquid whose effect predominates despite its much lower specific susceptibility.

The action on a paramagnetic liquid may be shown by dissolving enough ferric chloride in the alcohol or ether to color it a deep yellow. The bubble is now seen to move away from the magnet. It should be noted that if the alcohol is not anhydrous, some hydrolysis of the ferric chloride may occur with deposition of brown ferric hydroxide which obscures the walls of the tube. Ether is an excellent solvent for ferric chloride and does not contain enough water to hydrolize the salt. It is convenient to have two level tubes, one permanently sealed up with alcohol or ether and the other with an ether solution of the ferric chloride. These last indefinitely and are then always available to demonstrate diamagnetism and paramagnetism.

The arrangement shown in Fig. 5 may be used to project an enlarged image of the level tube on a screen. The level tube is placed in a rectangular box made of colorless Lucite. This box is filled with water, which reduces the cylindrical lens effect of the tube sufficiently to permit a clear image of the tube to be projected. Refraction effects cause the image of the air bubble to stand out sharply as a black cylinder with rounded ends. A simple lens of 10-cm focal length serves very well as a projection lens, giving an adequate image of the level tube magnified 20 to 30 times.

It would be convenient to have some sort of track or a swivel arrangement to guide the magnet to be placed around the Lucite box without bumping it and thereby disturbing the position of the bubble. But it is preferable to practice a little and use the magnet free-hand because it is easier to show that the bubble may apparently be "pulled" back and forth along the tube—or "pushed" in the case of a paramagnetic liquid. It is essential to mount the Lucite box securely so that it will not be knocked over at a critical moment. Provision must also be made conveniently to level the tube to bring the bubble somewhere near the center.

Change in address of the editorial office. Manuscripts for publication in the American Journal of Physics should be submitted henceforth to Dr. Thomas H. Osgood, Michigan State College, East Lansing, Michigan. Doctor Osgood, the new editor [see *Am. J. Physics* **16**, 125 (1948)] will assume responsibility for the Journal as soon as the somewhat involved procedure of transferring the editorial work has been completed.—D. R.

Some Improved Experiments for the Heat Laboratory

CHARLES WILLIAMSON AND W. H. MICHENER
Carnegie Institute of Technology, Pittsburgh 13, Pennsylvania

IN general physics laboratories the equipment for experiments in heat presents a difficult problem. Poor results and trouble with apparatus are quite prevalent. It is the opinion of the authors that some experiments should be carried out with crude apparatus when this is sufficient to illustrate the principles. But the student—particularly the engineering student—should also become acquainted with good instruments and good measuring technics. The one point on which all instructors will agree is that the apparatus should be trouble-free and student-proof, in so far as this is possible.

Thermal Expansion

The apparatus which we have developed for measuring the thermal expansivity of a brass tube is illustrated in Fig. 1. Its principal virtues are that it works well, gives good results, and satisfies the student. Micrometer microscopes were discarded from this experiment with some reluctance, since it was felt that students should learn how to use these devices. It was our experience, however, that few learned even enough to appreciate them. The measurements were poor, and the microscopes suffered many casualties. The use of a dial gage for measuring the change of length of the tube greatly simplifies the experiment. More readings can be taken, and a good curve of temperature *versus* expansion can be plotted.

By filling the system with ice water, one may readily produce an initial temperature between 0°C and 5°C. The maximum temperature should be slightly below the boiling temperature of the water to prevent scalding the operator. Readings may also be taken as the system cools—the cooling being hastened by adding crushed ice and draining off the excess water. (The drain consists of a side tube not shown in the diagram.) The readings can be made sufficiently precise to show that the cooling curve deviates very slightly from the heating curve because of the slow expansion of the wooden base.

The pump enables one to circulate heated water through the tube. When the flame is reduced or withdrawn, the water is quickly brought to a uniform temperature. The pump is somewhat novel in that it is purposely designed with considerable clearance between the piston and the cylinder walls. The down stroke still produces sufficient pressure to circulate water through the tube, and at the same time the leakage past the piston thoroughly stirs the water in the cylinder.

This type of pump has been found useful in other experiments. Some students would prefer an electrically operated pump, but this would be a very doubtful advantage to the instructor who is responsible for maintenance.

Specific Heat

The measurement of the specific heat of a metal specimen is a very simple operation, but the student is often disappointed with his result. One of the principal sources of error is the loss of heat from the specimen while it is being transferred to the calorimeter. The apparatus in Fig. 2 enables one to make this transfer easily and quickly. The heating chamber may be swung to a position directly over the calorimeter. The stirrer is a coarse wire basket used to catch the specimen and prevent splashing.

If the samples are to be supported in the heating chamber as illustrated, it is necessary to cut them to the required shape, bore a hole for a thermometer and a central hole and slots for the fiber supporting rod. This fiber rod carries a pin at the lower end. When the rod is turned till the pin matches the slot, the specimen drops. Two or more samples may be heated at once but dropped one at a time as desired. A large cork stopper (not shown) closes the lower end of the heating tube while the samples are being heated.

With this apparatus it is quite usual for students to get results that are apparently correct to within 1 or 2 percent, which are the limits of accuracy set by the thermometer readings.

Thermal Conductivity of Metals

There is considerable instructional value in a careful measurement of thermal conductivity be-

cause of both the theory and the technic involved. The apparatus shown in Fig. 3 is designed to eliminate the sources of most serious error, and its features can be seen easily by the student. Two specimens—brass and aluminum—are measured at the same time. There is no provision for exchanging samples.

Thermocouples are used to measure differences in temperature between pairs of points in the bars and also the rise in temperature of the circulating water which withdraws the heat. Double-pole switches are used to connect the couples in turn to a millivoltmeter (0 to 1 mv). In normal operation the temperature difference between the two points in the brass bar is about twice as large as that registered by the other couples. For this reason the other three couples are made double, that is, two couples in series [not as shown in Fig. 3*a*]. The calibration of the thermocouples is given on a chart posted near the apparatus. The observant student finds, however, that no calibration is necessary provided the couples are alike, since in the calculation one uses only the ratio of the rise in temperature of the cooling water to the temperature gradient along the bar. (The authors confess that they designed the apparatus and calibrated the thermocouples before they realized that the calibration was unnecessary.)

The holes in the test bars are made only large enough to accommodate the thermocouple wires, which are No. 30 copper and constantan. The diameter of the bars (about 2 in.) and the distances between couples can be measured by the student when the front panel is removed, as shown in Fig. 3*b*. This front panel has an attached bag of rock wool insulation.

The principal problem in operation is to adjust the rate of flow of the cooling water. When this is properly done, the rise in temperature of the cooling water is not too different from the temperature differences measured along the bars. An overflow vessel (not shown) is used to give a constant pressure head for this water. Reasonably steady conditions may be attained in 15 or 20 min after steam is admitted. It is interesting also to take temperature readings during this early period. For a run the cooling water is caught for a measured time of 10 min during which all temperatures are read periodically (with the hope that they will remain constant).

One thermocouple is used to measure the temperature drop at the junction of the two bars. This difference is, to most students, surprisingly large. The observation may show the student why he cannot assume that the end of the bar which projects into the steam chamber is at the temperature of the steam, and the opposite end of the other bar at the average temperature of the cooling water.

In making the specimens, the ends to be joined were merely faced in a lathe and then held together with a short headless screw. Better thermal contact might have been made by other methods. We decided, however, that we did not need good contact, but rather that the contact should be uniform over the whole area. Moreover, small variations in a poor contact would cause

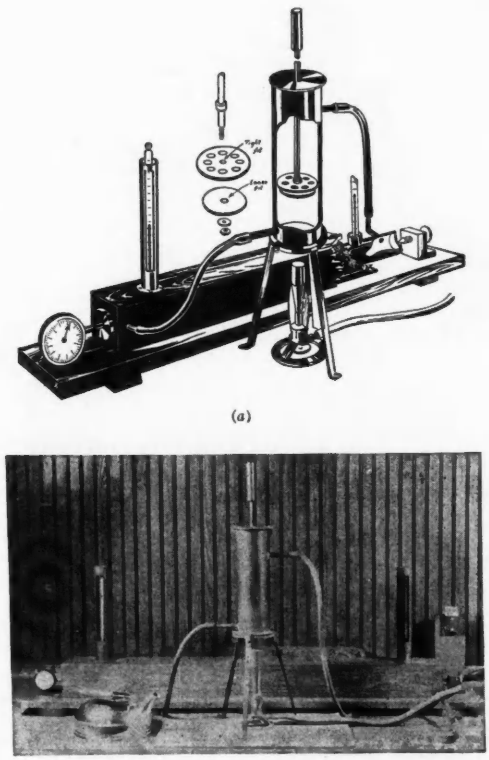


FIG. 1. Thermal-expansion apparatus: (*a*) drawing showing construction; (*b*) photograph of assembled equipment.

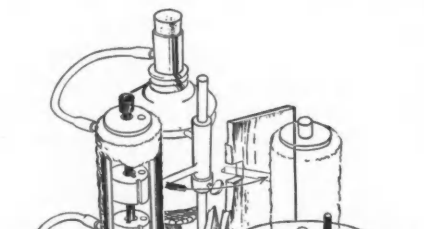
less trouble than the same variations in a good contact.

If we were rebuilding this apparatus we would perhaps choose as one of the specimens a pure metal, such as copper, whose thermal conductivity is well known and would serve as a reference. A copper bar would need to be longer, however, to give a sufficient temperature difference to measure. The commercial aluminum which we used has a thermal conductivity scarcely two-thirds that of pure aluminum. We think it is desirable that one sample (but perhaps not both) yield a value which cannot be checked in a table.

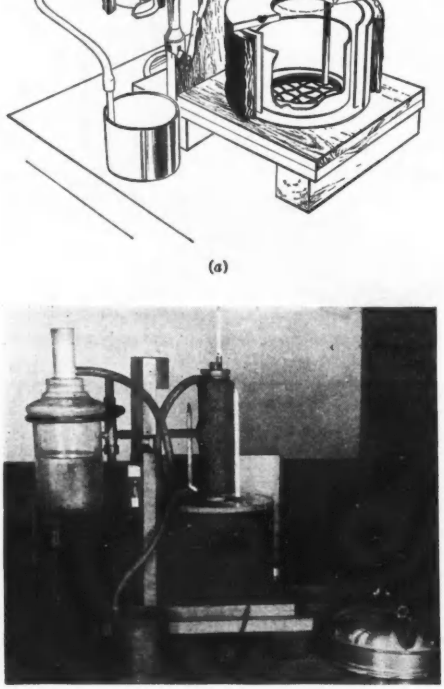
The Pyrex boiler is conventional, but it has some modifications which may be of interest. The sheet-iron chimney surrounding the boiler is well worth providing. It not only makes the heating much more efficient, but it also makes it quite constant even if there is considerable breeze. The boiler may be filled without unscrewing the top (which usually sticks). The large cork stopper is removed and water poured in from a teakettle. (A few small copper teakettles about the laboratory are very useful.)

Thermal Conductivity of Asbestos

We are not supplying a photograph of this apparatus, since we are not very proud of its appearance. Our first crude experimental model worked so well that we have not rebuilt it. Some of the details can be seen in Fig. 4. Asbestos paper or sheet is wrapped around a thin-walled brass tube about 1 m long until the total thickness of asbestos is between 1 and 2 cm. A sheet of

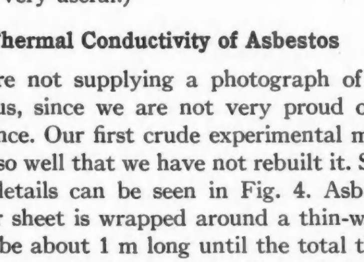


(a)

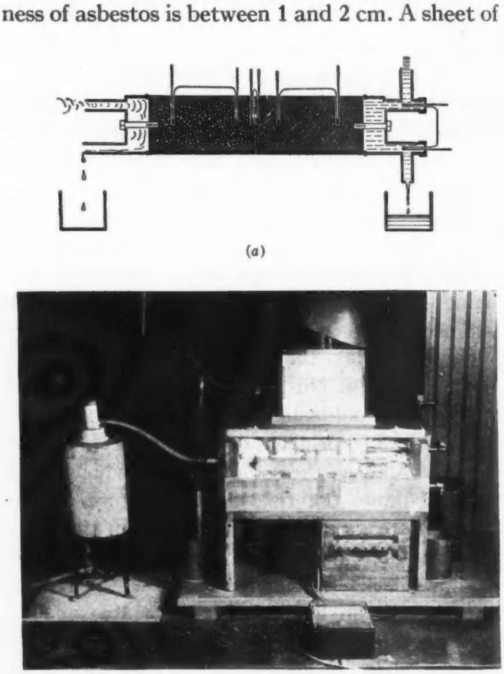


(b)

FIG. 2. Specific-heat apparatus: (a) drawing showing construction and operation; (b) photograph.



(a)



(b)

FIG. 3. Thermal-conductivity apparatus: (a) cross-sectional drawing; (b) photograph, with front panel of surrounding box removed.

copper is wrapped around the central half and soldered. This copper insures a more uniform surface temperature.

Temperatures are measured by six copper-constantan thermocouples which are built into the apparatus. The hot junctions are placed somewhat as indicated. Three of the couples are next to the brass tube (except for electrical insulation) and three outside the asbestos. The wires are led out longitudinally so that there will be no temperature gradient near the junctions. The copper wires each lead to a switch, beyond which they are joined and led through a millivoltmeter to a common cold junction which may be immersed in a jar of crushed ice.

The heating element consists of a porcelain tube with a close-wound single layer of No. 22 cotton-covered copper wire. Near the ends, to compensate for heat losses, the wire is wound in a double layer. This rather unconventional heating element is easily made and works satisfactorily for moderate temperatures. We expect eventually to replace it with bare resistance wire wound on a tube that is properly grooved to insure even spacing. The heater tube is wrapped with tape near each end to make it fit snugly inside the brass tube. The ends are corked to prevent air circulation.

The current is measured with an ammeter. Voltmeter leads are soldered to the heater wire at points 20 cm apart near the middle of the heater so that one can measure the power supplied to a known length of tube. The current is controlled by two 100-w lamps (covered by a metal shield to reduce eye strain). For 10 or 15 min of preliminary heating, both lamps are turned on. After this, one lamp is used. When the apparatus is used daily we often leave the one lamp on day and night. The principal difficulties in obtaining steady-state conditions arise from voltage fluctuations and from changes in temperature and air movement in the laboratory. Typical readings are 12.2 v, 0.65 amp, 75°C and 55°C. The temperature of the copper windings is probably about 80°C.

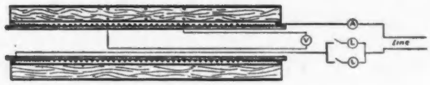


FIG. 4. Diagram of apparatus for measuring the thermal conductivity of asbestos.

In this apparatus it is obviously essential that the heat flow be radial. If there is any longitudinal heat flow, there will be some longitudinal temperature gradient. The thermocouples indicate that this is not appreciable. Our guess of the number of extra turns of wire to be added at the ends of the heater was apparently good enough. There is sometimes a slight difference indicated by the three outer couples. This presumably results from air currents. The three inner couples seldom show an appreciable difference.

We are indebted to our colleague, Russell T. Hyde, of the Department of Painting and Design, for the drawings illustrating this article; to Oscar G. Fryer, now of Drury College, for the design of the specific-heat apparatus; and to our chief mechanician, Clayton I. Brown, for his careful construction work and for assistance with many details.

The really great school and college teachers are not primarily teachers of biology, English or economics. They are teachers of young men and women. Their success can be measured by the degree to which they correct, humanize and enrich the student's perspective, and give him wider interests, new horizons, enlarged frames of reference, and those sounder habits of working and thinking which make it possible for him to discover the relevant facts in any field, and in his own reach valid conclusions.—CHRISTIAN GAUSS.

American Standard Letter Symbols for Physics

FINAL REPORT (No. 4) OF THE COMMITTEE ON LETTER SYMBOLS

THE last printed report of the Committee on Letter Symbols appeared in 1940.[1] The war interfered with the work of the Committee; but, in the middle of 1945, activities were resumed, and the Committee's final report is presented herewith.

The preparation of this report has entailed a very careful study of every comment that was received concerning the preceding report.[1] In several instances, elaborate lists were made of symbol usage in special fields in order to guide our recommendations. We owe thanks to many Association members who, with no thought of credit, helped us to decide complex cases.

In addition to a recommended list of letter symbols, this report contains the general and special principles, slightly modified since 1940, that have guided the Committee's work, together with a discussion of certain controversial symbols. There is also a statement of the relation of the Association Committee to the American Standards Association, from which the same personnel hold appointment.

The list of symbols given in the preceding report,[1] arranged alphabetically by symbol, has been included in recent editions of the *Handbook of Chemistry and Physics*.[2] The present American Standard will be published in future editions.

General Principles of Letter Symbol Standardization

As a guide in its work and as one means of achieving uniformity with other symbol committees, we have adopted the following "General Principles." The committee helped to formulate these principles and urges their acceptance by the members of the Association.

1. Manuscript. In preparing manuscripts, it is suggested that authors give careful attention to the use of symbols from this and other standard lists and to the principles here given. Symbols used should be defined clearly. When a table of symbols is not given, it is desirable to make reference to the standard lists from which the symbols are taken. The many numbers, letters, and signs that are similar in appearance should be distinguished carefully.

2. Letter symbols. A letter symbol is a single character, with subscript or superscript if required, used to designate a physical magnitude in mathematical equations and expressions. (For the definition of "physical magnitude," see Special Principle 9.) Two or more symbols together always represent a product.

Letter symbols are to be distinguished from abbreviations, mathematical signs and operators, graphical symbols, and chemical symbols:

(*a*) ABBREVIATIONS are shortened forms of names and expressions employed in texts and tabulations and should not be used as symbols in equations.

(*b*) MATHEMATICAL SIGNS AND OPERATORS are characters used with letter symbols to denote mathematical operations and relations.

(*c*) GRAPHICAL SYMBOLS are conventionalized diagrams and letters used on plans and drawings.

(*d*) CHEMICAL SYMBOLS are letters and other characters designating chemical elements and groups.

3. The same symbol should be used for the *same physical magnitude* regardless of the units employed and regardless of special values occurring for different states, points, parts, times, etc. Units or special values may be distinguished when necessary by subscripts, superscripts, or by upper- and lower-case letters when both are specifically included as symbols in a standard list. The units used should be indicated when necessary. Sometimes different symbols are used for the components of a vector. (See also Special Principle 9.)

4. Subscripts. A subscript preferably should be a single character. It is commonly employed to indicate constancy of a particular physical magnitude, such as pressure or temperature, when there are other variables. A multiple subscript, sometimes divided by a comma, refers to more than one state, point, part, time, etc. A subscript should not be attached to a subscript. Further uses of subscripts are listed in Principles 3 and 6.

5. Superscripts. A symbol with a superscript, such as a prime (′) or a second (″), should be enclosed in parentheses, braces, or brackets before affixing an exponent. A complicated exponent (or any other expression frequently repeated) may be replaced by a single symbol selected to represent it. Reference marks should not be attached to symbols. Further uses of superscripts are listed in Principles 3 and 6.

6. Conflicts. Conflicts which would occur when different physical magnitudes are assigned the identical symbol in the same or different standard symbol lists may be resolved in one of the following ways:

[1] "Proposal to standardize letter symbols, Report No. 3 of the Committee on Letter Symbols and Abbreviations," *Am. J. Physics* **8**, 300 (1940).

[2] C. D. Hodgman, ed., *Handbook of chemistry and physics* (Chemical Rubber Publishing Co., annual editions).

(a) For one or more of the conflicting uses, the given symbol may be employed with subscript or superscript selected by the author.
(b) If one of the magnitudes has an alternate symbol in a standard list, it may be used.
(c) A slight change in the name of the magnitude may remove the conflict. For instance, one may use L for "length of radius" when r for "radius" conflicts with r used for another magnitude.

7. Unlisted magnitudes. To symbolize a special value of a listed magnitude, see Principle 3. The symbol chosen by an author for a physical magnitude not appearing in any standard list should be one that does not have already a different meaning in the field of the text.

8. Typography. Letter symbols, letter subscripts, and letter superscripts, whether upper or lower case, should be printed with italic type unless definitely specified otherwise. On manuscript this is indicated by underlining each symbol that is to be italicized. Special types, such as Old English and type that is currently used for vector magnitudes, should be avoided for scalar magnitudes. When special type is used for vector magnitudes, the same italic letter should be used for the corresponding scalar magnitude. Vertical Arabic numerals should be used as coefficients in equations and in subscripts and exponents. Bars, dots, and other modifying signs and operators should be used in the manner currently recognized by mathematicians.

Special Principles for Use in Physics

The following additional principles relating more especially to symbols in physics have been adopted by the Committee.

9. It is understood that all letter symbols, when not operators, represent physical magnitudes; that is, the symbol is equivalent to the product of a pure numeric and units. The numeric designates the measure of the magnitude and is always the modulus value in the case of fixed constants. Thus, e is the symbol for the charge of the positron, and $-e$ the symbol for the electronic charge. The same symbol is employed regardless of units. Where it is necessary to indicate the units for a magnitude or the system of units for an equation, subscripts are often convenient or the reference number of the equation may carry in addition to the numerals a letter designating the unit system. Thus, m, s, and p may indicate the electromagnetic, the electrostatic, and the practical (mks) system of units, respectively. For example, all quantities in an equation which is designated as Eq. (6p) are in practical units.

10. Vectors and dyadics. Vectors and dyadics should be printed in boldface roman type. On a manuscript, this is shown by placing a wavy line under each such symbol. On the blackboard, double-lined letters or an arrow above each symbol or a wavy line beneath each symbol may be used.

11. Cartesian coordinate system. The Cartesian coordinate system should preferably be right-handed and the axes identified by the capital letters X, Y, and Z in vertical roman type. No one orientation of the axes is proposed.

12. Modifying signs. The following modifying signs are attached to physics symbols.

Magnitude	Modifying Sign	Example
Average value	Overbar	$\bar{a}$
Initial value	Subscript zero	v_0
Principal value (optics)	Overbar	$\bar{\varphi}$
Standard or reference value	Subscript zero	g_0

13. The operator Δ preceding a letter symbol means the change in the value of one variable. It should not be employed to symbolize the difference between two independent variables.

14. Use ΔH or Δh, depending upon the quantity of material, for all heats of reaction and phase change (fusion, evaporation, sublimation, etc.). A single letter to represent these heats is sanctioned, but not recommended. This single letter must not be H or h. Either letters or numbers may be employed as subscripts for either components or phases. To avoid multiple subscripts, Roman numeral superscripts (for phases only) may be employed.

Recently, it has become apparent that there is not complete agreement upon the first sentence of Special Principle 9. About 20 years ago, a joint committee[3] of physicists and engineers in Germany reported that the symbols employed in equations could be interpreted as designating either numbers or the physical magnitudes themselves (numbers and units). Though some writers make a distinction, "quantity" and "magnitude" are generally used interchangeably. As an alternate to the first sentence of Special Principle 9, one might then write:

> It is understood that all letter symbols, when not operators, may represent either the measures of physical magnitudes or the magnitudes themselves. On the second interpretation, which is widely employed, the symbol is equivalent to the product of a numeric and a unit.

This matter is not the concern of the Committee on Letter Symbols, and our only action on it is to call the problem to the attention of readers.

The American Standards Association

The American Standards Association was organized in 1918 to serve as the clearing house for industrial, technical and governmental groups to develop and coordinate their standardization

[3] The committee which prepared the report consisted of J. Wallot, F. Emde, H. Diesselhorst, G. Hamel, W. Kösters, G. Madelung, H. Reissner and K. Scheel. *Elektrotech. Z.* **51**, 1, 586 (1930).

programs. It is a federation of approximately 100 national organizations and some 2000 Company Members. The chief function of the ASA is to provide systematic means for the establishment of American Standards. To this end, it brings together manufacturers, distributors, consumers, technical specialists, and others directly concerned. Usually, one of the cooperating bodies acts as sponsor of the Standard. The ASA ascertains whether a preponderance of these interests desires a national standard on the subject, and it provides for the organization of a balanced committee consisting of official representatives of the bodies directly interested. When the committee has reached substantial agreement in regard to the provisions of the proposed standard and the Association is assured that the standard represents a truly national consensus, the ASA approves it as an "American Standard."[4]

The same personnel that constitute the Association's Committee on Letter Symbols holds appointment from the ASA as its Subcommittee No. 10, Letter Symbols for Physics, of Sectional Committee Z 10 on Letter Symbols and Abbreviations for Science and Engineering. The organization of Committee Z 10 is as follows:

Chairman. H. M. Turner, Department of Electrical Engineering, Yale University, New Haven, Conn.

Secretary. W. H. Deacy, Sr., American Standards Association, 70 East 45 St., New York 17, N. Y.

Subcommittees and their chairmen

1. Letter symbols and signs for mathematics. Albert A. Bennett, Brown University, Providence 12, R. I.

2. Symbols for hydraulics. (No chairman at present.)

3. Symbols for mechanics. R. E. Peterson, Westinghouse Research Laboratories, East Pittsburgh, Pa.

4. Symbols for structural analysis. Albert Haertlein, Harvard University, Cambridge, Mass.

5. Symbols for heat and thermodynamics. L. C. Lichty, Yale University, New Haven, Conn.

6. Symbols for illuminating engineering. A. E. Parker, Worcester Polytechnic Institute, Worcester, Mass.

7. Letter symbols for aeronautics and aerodynamics. T. F. Ball, Johns Hopkins University, Baltimore, Md. *Secretary,* R. E. Hopgood, 420 Lexington Ave., New York 17, N. Y.

8. Letter symbols for electrical quantities. Edward Bennett, University of Wisconsin, Madison, Wis.

9. Symbols for radio. H. M. Turner, Yale University, New Haven, Conn.

10. Symbols for physics. Harold K. Hughes, Socony-Vacuum Laboratories, Brooklyn 22, N. Y.

11. Abbreviations for scientific and engineering terms. George A. Stetson, American Society of Mechanical Engineers, 29 West 39 St., New York 1, N. Y.

12. Letter symbols for chemical engineering. John H. Perry, E. I. du Pont deNemours Co., Wilmington, Del.

In addition to Committe Z 10, the American Standards Association has two other committees charged with symbol standardization. These deal with gears (B 6) and with illuminating engineering (Z 7).

For a short history of letter-symbol standardization in the ASA, the reader is referred to reference 1. Sectional Committee Z 10 was organized under the procedure of the ASA in January 1926, with the American Association for the Advancement of Science, the American Society of Civil Engineers, the American Institute of Electrical Engineers, the American Society for Engineering Education and the American Society of Mechanical Engineers as joint sponsors. The Committee was reorganized in October 1935. It now consists of representatives of 36 national societies, associations and governmental departments.

Following approval by the members of Committee Z 10, voted by letter ballot, and subsequent approval by the sponsor organizations, the present standard was submitted to the ASA for approval and designation as an American Standard. This designation is expected to take place early in 1948.

Publications of Other Symbol Committees

Because physics covers such a wide field, the Committee has endeavored to keep itself informed on the work of all other symbol committees and to cooperate with them in choosing uniform symbols wherever possible. The following publications in this field are available at present.[5]

1. "American Standard Abbreviations for Scientific and Engineering Terms," ASA Z 10.1 (1941). To be revised in 1948 or 1949. The Association also has a Committee on

[4] This paragraph has been adapted from ASA publication PR27, "The organization and work of ASA sectional committees" (1947).

[5] Report No. 8 may be obtained from The Chemical Society, Burlington House, Piccadilly, London, W.1; all the other publications are distributed by the American Standards Association, 70 East 45 Street, New York 17, N. Y.

Nomenclature and Abbreviations under the chairmanship of Duane Roller.

2. "American Standard Letter Symbols for Hydraulics," ASA Z 10.2 (1942).

3. "American Standard Letter Symbols for Mechanics of Solid Bodies," ASA Z 10.3 (1942).

4. "American Standard Letter Symbols for Heat and Thermodynamics, Including Heat Flow," ASA Z 10.4 (1943); see also *Am. J. Physics* **11**, 344 (1943).

5. "American Standard Letter Symbols for Chemical Engineering," ASA Z 10.12 (1946).

6. "Illuminating Engineering Nomenclature and Photometric Standards," ASA Z 7.1 (1942).

7. "American Standard Letter Symbols for Gear Engineering," ASA B 6.5 (1943).

8. "Report of a Joint Committee of The Chemical Society, The Faraday Society and The Physical Society on Symbols for Thermodynamical and Physico-Chemical Quantities and Conventions Relating to Their Use."

9. "International Electrotechnical Letter Symbols," U. S. National Committee of the International Electrotechnical Commission, USNC 67 (1946).

International Cooperation[4]

All industrial countries have found it necessary to set up national standardizing bodies. The ASA maintains cooperative relations with them and maintains complete files of their standards. Much of this cooperation is carried on through the International Organization for Standardization (commonly known as ISO) and through the International Electrotechnical Commission (IEC). The U.S. National Committee of the IEC (USNC) is affiliated with the ASA. In 1939, 27 countries were affiliated with the IEC and technical undertakings were in progress on 28 subjects. The United States was participating in all of these projects and was taking the leadership in five of them. In this connection, the reader is referred to publications 8 and 9 in the preceding section of the present report. Our committee cooperated to some extent in the international standardization of letter symbols prior to World War II.

The Problems of Symbol Standardization

At the outset, the Committee recognized that it faced a number of difficult problems, the handling of which would eventually determine the success or failure of its work. While these problems were studied only in relation to the primary assignment, it may be of some value to others doing similar committee work to discuss some of them here.

In any controversial field, it is essential to provide means for the full exchange of facts and opinions. Committee members and their correspondents would be quick to resent, and rightly so, any attempt on the part of the chairman to utilize his position for the furtherance of his own views. This is not to say that a chairman should be no more than a recording secretary for, presumably, he has been chosen for his interest and knowledge in the field and a willingness to study it intensively. This alone would entitle him to an opinion. Every member of the present committee has been kept fully informed of all the views expressed by his fellow members and by others commenting upon its reports. In this way, he has always known the reasons for every decision.

A vote has been taken on every item in this report. Over 75 percent of the symbols proved to be noncontroversial and were readily agreed to. Where the Committee could not come to a unanimous decision on the choice of a symbol, further discussion ensued. Generally, agreement was eventually reached, but in some few instances it was decided to place the quantity in the Supplementary List without a recommendation.

No small part of the work has been the give-and-take with other ASA subcommittees. One of our basic desires has been to achieve uniform lists with our fellow members of Z 10. We are pleased with the result of this cooperation and believe that the effort was well worth while.

Much thought has been given to the question of how far to go in standardization. In this we have tried to steer a middle course. Included are all the common terms in elementary physics and some in advanced physics and related fields. Most symbols in mathematics, crystallography, meteorology, x-rays, statistics and so forth, have been omitted. The section of the Committee's master file labeled "morgue" is nearly as extensive as the recommended list. Some of the decisions to omit or include quantities were admittedly arbitrary and were dictated, in part, by practical considerations. No attempt has been made to standardize symbols for a subject in process of formation, even though, as in the case of nuclear physics, parts of it have crept into recent elementary texts.

There has been general agreement among

physicists that some measure of standardization is desirable. One notable exception was an eminent author who disagreed in principle with the basic objective of the Committee and thought "it would be very well were the work of such committees allowed to remain buried in the obscurity in which they were written." At the other extreme stood the late Dr. Sanford A. Moss, whose enthusiasm extended to attempts at international standardization. As mentioned elsewhere in this report, the International Electrotechnical Commission is the only agency which has so far produced standard symbols on a world basis. In these matters, too, the Committee has resisted pressures to take an extreme position. We feel that much can be accomplished through a study of the United States field first and have left international standardization to our successors.

Symbol standardization has its amusing moments, too, as when another outstanding physicist wrote: "In studying over your report, I note the statement on page 315 [Report No. 3] that your committee has attempted to function as a 'melting pot', and I suppose this necessarily implies that the committee is in a 'hot spot' in trying to decide many of these questions." On another occasion, a jovial correspondent objected to our approval of alternate symbols by stating that "its relation to standardization is the same as the relation of two pints of whiskey to clear vision; they create two images."

One cannot consider symbolism without becoming aware of nomenclature.[6] To all attempts —and there have been many—to introduce the standardization of terms into our work, we have steadfastly paid no heed. The Association has a duly appointed Committee on Terminology and Abbreviations to which all such matters should be referred. The recommended list is carefully cross-indexed under all common synonyms. As an illustration of the manner in which the Committee has avoided a similar decision on unit systems, the reader may consult the symbols for capacitivity, permittivity and dielectric coefficient.

The Controversial Electrical Group of Symbols

As a final illustration of the problems of standardization, consider the group of electrical symbols which have, without doubt, been the most difficult to choose. Though small in number, these have received a disproportionate share of attention by the Committee. The steps that led to our final recommendations are illustrative of the careful weighing of conflicting demands, which was also done in many other cases.

Contributing to produce the divergences in usage are three factors: (i) physicists and engineers use some of the quantities in different contexts; (ii) even among physicists, the meaning of some of these is moot; (iii) for the most part, neither physicists nor engineers appreciate each other's needs, or the need to choose a group of symbols simultaneously. The development of radar during the war, principally by physicists, has had a genuinely salutary effect on this situation.

Table I lists this group of quantities, the various proposals for symbols made to us, and also the Committee's recommendations. Only a little consideration is needed to recognize the many relations among these quantities that make separate symbols desirable for each of them. Further study shows that this is practically impossible, considering the extent to which some symbols are already in use and the limited number of symbols available. Some examples follow.

TABLE I. Proposed and recommended symbols in electricity.

Quantity	Symbols Proposed to Committee	American Standard Symbols
Area	A, S, σ	A, S, σ
Current density	A, a, i, j, J, S, u	J
Electric conductivity	σ, γ, κ	$\sigma, (\gamma)$
Electric dipole moment	p, μ, μ_e	μ, μ_e
Electric field strength	$X, E, \mathcal{E}, K, F$	E
Electric moment	p, μ	p
Electric potential	E, V, v	V
Electric susceptibility	η, ϵ	η
Electromotive force (emf)	$E, \mathcal{E}, V, \mathfrak{F}$	$\mathcal{E}, (E)$
Energy	u, U, E, W, T	See main list
Magnetic dipole moment	μ, μ_m	μ, μ_m
Magnetic moment	μ, m	m
Magnetic pole strength	p	p
Magnetic susceptibility	k, K, κ, χ	k
Magnetization	M, J, I	M
Moment of inertia	I, K, J	$I, (J)$
Naperian base	e, ϵ, exp	e, ϵ
Permittivity (dielec. coef.)	k, K, ϵ, κ	ϵ
Velocity	C, c, u, v, w, α	See main list
Volume	V, v, τ	$V, (\tau)$
Weight (gravitational force)	w, W, G, F	$w, F, (W)$
Work	w, W	$W, (w)$

[6] For example, several commentators have discussed our handling of the term "voltage"; see also *Am. J. Physics* **14**, 340 (1946); **15**, 191 (1947); **15**, 428 (1947).

The symbol for area should be distinct from those for current density, Poynting's vector, and the Hertz vector. Electric conductivity and surface electric charge density may occur together, while in electrical engineering it is essential that conductivity and propagation constant be distinguished. The definition of the latter in terms of the attenuation and phase constants, α and β, namely,

$$\gamma=\alpha+j\beta,$$

is so well established that γ cannot be chosen as the primary symbol for conductivity even though the Committee has found it in wide use. Because it is so difficult to distinguish κ from an italic k, the use of the former for any quantity is deprecated. R. T. Birge,[7] the Committee on Electric and Magnetic Units,[8] and others have emphasized the distinction between permittivity and dielectric coefficient which, in the mks system of units, results in the assignment of different values to these two quantities. Electric dipole moment must be distinguished from total electric moment, polarization and electric quantum number. Similarly, magnetic dipole moment, magnetization and total magnetic moment should have different symbols. There may possibly be no objection to letting μ stand for permeability and both dipole moments, since these rarely occur in the same equation. Permittivity, electron charge, naperian base and electric field strength occur together in equations for a leaky condenser, in electron optics and elsewhere.

These electrical quantities are so important that the Committee was loath to avoid the issue by placing them in its supplementary list. Therefore, a meticulous survey was made of the usage in all well-known books, experts in the field were consulted and, finally, all of these *facts* were collected in a ten-page "Revision C," together with the recommendations of other committees and of all our correspondents. Rather surprisingly, when all the evidence was before us, it was found that most of the choices were obvious.

While use of this list may produce some changes, we believe that we have studied the controversial electrical group of symbols as thoroughly as any committee can. With the exception of the use of K for electric field strength by Subcommittee No. 8 on Electric Quantities, we are pleased to report full agreement on this list by all subcommittees.

In view of the Committee's expressed willingness to compromise for the purpose of achieving uniform lists, it may be well to emphasize at this point that not all of the "give" has been on the part of physicists. Our fellow ASA committees have been equally willing to accede to some of our insistent demands for change.

[7] *Am. J. Physics* **2**, 41 (1934).
[8] *Am. J. Physics* **6**, 144 (1938).

American Standard Letter Symbols for Physics

The list of symbols given here embodies the final recommendations of our committee and will soon be designated as the American Standard by the ASA.[9] In this list, symbols separated by commas are alternates of equal rank; reserve symbols, the use of which is discouraged, are given in parentheses.

absolute humidity . . . . . . ρ
absolute temperature . . . . . . T, (Θ)
absorption coefficient for a room, mean (*acous.*) . . . . . . a
absorption coefficient of absorbing material (*acous.*) . . . . . . α
absorption index $[=(\lambda/4\pi x)\ln P_0/P]$ . . . . . . κ
acceleration, angular . . . . . . $\boldsymbol{\alpha}$
acceleration, gravitational . . . . . . g
  local value . . . . . . g_L
  standard value [=980.665 cm sec^{-2}] . . . . . . g_0
acceleration, linear . . . . . . $\mathbf{a}$
acceleration of free fall . . . . . . $-g$
accommodation coefficient . . . . . . a
acoustic conductivity of an opening . . . . . . σ, (c)
acoustic impedance, specific $[=ZA]$ . . . . . . z
acoustic intensity (e.g., erg cm^{-2} sec^{-1}) I
acoustic reactance, specific $[=XA]$ x
acoustic resistance . . . . . . R
acoustic resistance, specific $[=RA]$ . . . . . . r
acoustic source, strength of simple . . . . . . A
action . . . . . . S
action variable . . . . . . J
activity at time t (*radioactivity*) I
activity (*chem.*) . . . . . . a
activity, initial (*radioactivity*) . . . . . . I_0
adiabatic exponent $[pV^\gamma=\text{const.}]$ . . . . . . γ
admittance (reciprocal impedance) . . . . . . Y
admittance with plate load, grid or imput
  $[=g_g-jb_g]$ . . . . . . y_g
admittance, output (plate) $[=g_p-jb_p]$ . . . . . . y_p
affinity, electron $[=w/e$, volts$]$ . . . . . . φ
affinity, gross electron . . . . . . φ_g
altitude . . . . . . h, y
amplification factor $[=-(\partial e_p/\partial e_g)_{i_p}]$ . . . . . . μ
amplification factor, gas (*photoelectric tube*) . . . . . . μ
amplification of amplifier, power . . . . . . A, A_p
amplification of amplifier, voltage . . . . . . A, A_v
amplitude . . . . . . A
amplitude of simple harmonic pressure (*acous.*) . . . . . . P
angle, azimuth (*optics*) . . . . . . ψ

[9] This final list, rearranged by symbol, may be found in the *Handbook of chemistry and physics* (Chemical Rubber Publishing Co. annual editions. The "American Standard letter symbols for physics" may also be purchased from the ASA.

angle of incidence or angle between ray and normal in first medium	φ, i
angle of refraction, or angle between ray and normal in second medium	φ', r
angle, critical	φ_c
angle, epoch	$\epsilon, (\varphi)$
angle, glancing	θ
angle of contact	θ
angle of deviation	δ
angle of diffraction	θ
angle of incidence, principal	$\bar{\varphi}$
angle of minimum deviation	D, δ_m
angle of optical rotation	α
angle of reflection	r
angle of slope in image space	u'
in object space	u
angle, phase	φ
angle, plane	$\theta, (\alpha, \beta)$
angle, polarizing, or principal angle of incidence in dielectrics	$\bar{\varphi}$
angle, principal azimuth	$\bar{\psi}$
angle, refracting, of prism	A
angle, solid	ω
angular acceleration	$\boldsymbol{\alpha}$
angular dispersion [$=d\theta/d\lambda$]	D
angular displacement	θ
angular distance	θ
angular frequency of free vibration with damping	$\omega', (n')$
angular frequency of impressed force	$\omega, (p)$
angular frequency without damping	$\omega, (n)$
angular frequency, or periodicity, or angular velocity [$=2\pi f$]	ω
angular magnification [$=\tan U'/\tan U$]	γ
angular resolving power of telescope	α
angular speed	ω
angular velocity	$\boldsymbol{\omega}$
aperture (*optics*)	a
arc length	s
area	A, S, σ
area, moment of [$=\int y dA$]	Q
area of cross section, walls (*acous.*)	S
atomic number	Z
atomic volume	V
atomic weight	A, M
attenuation, optical	D
attenuation constant	α
Avogadro number (number of molecules per mole)	N_0
azimuth angle (*optics*)	ψ
azimuth angle, principal	$\bar{\psi}$
base of natural logarithms [$=2.718\ldots$]	ϵ, e
beam deflection or sag	δ
Bohr magneton [$=eh/4\pi mc$]	μ_0
Bohr radius [$=h^2/4\pi^2me^2$]	a_1
Boltzmann constant	k
Boltzmann function [$=\sum_i n_i \ln n_i$]	H
Bragg planes in a crystal, spacing of	d
Bragg reflection indices	$\begin{cases} h_1[=nh] \\ h_2[=nk] \\ h_3[=nl] \end{cases}$
breadth or width	b
brightness, or luminance [$=dI/dA \cos\theta$]	B
candlepower, or luminous intensity [$dF/d\omega$]	I
capacitance, or permittance	C
capacitance, partial (with suitable subscripts)	c
capacitance, reciprocal, or elastance	S
capacitive reactance	X_c
capacitivity, or dielectric coefficient (dielectric constant)	
In cgs units	ϵ
In mks units (*see* permittivity)	ϵ_r
Cauchy constants [$n=A+B/\lambda^2+C/\lambda^4$]	A, B, C
charge density, linear	λ
charge density, surface	σ
charge density, volume	ρ
charge, electronic	$-e$
charge, electric	q, Q
chemical potential (*see* Gibbs function)	μ
circle of least confusion, radius of	Z
circular frequency [$=2\pi f$]	ω
coefficient of accommodation	a
coefficient of friction	μ
rolling	μ_r
sliding, or kinetic	$\mu, (\mu_k)$
starting, or static (use f when μ designates the Poisson ratio)	μ_s
coefficient of recombination	α
coefficient of resistance, temperature	α
coefficient of restitution or of resilience	e
coefficient, potential	p
collision diameter of molecule	σ
collision frequency, molecular (no./time)	Z
compliance	C
compressibility factor (coefficient of compressibility; reciprocal of volume modulus of elasticity)	k
concentration (*solutions*), or use bracket [] notation	c
concentration, molecular, or molecular density [$=N/V$]	n
at 0°C and 1 atm (Loschmidt number)	n_0
condensation (*acous.*)	s
conductance, electric	G, g
conductance, grid or input	g_g
conductance, grid, at zero frequency and constant plate potential [$=(\partial i_g/\partial v_g)_{v_p}$]	k_g
conductance, plate or output	g_p
conductance, plate, at zero frequency and constant grid potential [$=(\partial i_p/\partial v_p)_{v_g}$]	k_p
conduction current	I
conductivity, electric	$\sigma, (\gamma)$
conductivity, equivalent (mho/equivalent)	Λ
conductivity, molecular (mho/mole)	μ
conductivity of an opening, acoustic	$\sigma, (c)$
conductivity, thermal	k
configuration-space volume	V
contact, or Volta, potential	V_v
contrast of photographic emulsion	γ
control ratio, grid (*thyratrons*)	μ
convection current	$I, \rho v$
critical angle	φ_c
critical pressure	p_c

critical temperature ... t_c, T_c
critical volume ... V_c
crystal planes, spacing of Bragg ... d
current, average ... I_{av}, $\bar{I}$
current, conduction ... I
current, convection ... I, ρv
current density, electric ... **J**
current, displacement ... $\dot{\mathbf{D}}$, $\dot{\mathbf{D}}/4\pi$
current, effective (rms) ... I
current, instantaneous ... i, (I)
current, maximum ... I_m, (I_{max})
current, maximum peak ... I_m, I_{mp}
current peak ... I_p, I, (I_{pk})
current, quiescent ... $\bar{I}$
current, rms or effective ... I
current, saturation ... I_s
current, steady, direct ... I
curvature ... K
curvature, radius of ... ρ

Debye characteristic temperature ... Θ
decay constant $[N=N_0 \exp(-\lambda t)]$ ... λ
decay modulus $[s=A \cos(\omega t+\varphi) \exp(-t/\tau)]$ ... τ
deflection of beam, galvanometer, etc. ... δ
degeneracy (statistical weight) ... g
degree of dissociation (*electrolytes*) ... α
degree of hydrolysis (*electrolytes*) ... h
degree of ionization (*electrolytes*) ... α
degrees of freedom (*kinetic theory*; *Gibbs phase rule*) ... f
density $[=m/V]$ ... ρ, (D)
density, linear (mass/length) ... λ, (ρ)
density, molecular or molecular concentration $[=N/V]$ ... n
at 0°C and 1 atm (Loschmidt number) ... n_0
density of electric charge, volume ... ρ
density of electric flux $[=d\Psi/dA]$ ... **D**
density of magnetic flux ... **B**
density, optical $[=\log_{10}P_0/P]$ ... D
density, surface $[m/A]$ ... σ, (ρ)
density, vapor ... ρ
depth and height ... h, y
deviation ... δ
deviation angle ... δ
deviation angle, minimum ... D, δ_m
deviation, frequency $[=\Delta f/f_r]$ ... δ
dew-point temperature ... τ
diameter ... D
diameter of molecule, collision ... σ
dielectric coefficient, or capacitivity (*see* permittivity) ... ϵ
dielectric flux density $[\Psi=\int \mathbf{D}\cdot d\mathbf{A}]$ ... **D**
dielectric flux, total $[=\int \mathbf{D}\cdot d\mathbf{A}]$ ... Ψ
difference in optical path ... Δ
difference in phase (e.g. cm) ... δ
diffraction angle ... θ
diffusion, coefficient of (*fluid*) ... D, (μ)
dioptric power $[=n/f]$ ... D
Dirac $\hbar[=h/2\pi]$ ... $\hbar$
direction cosines ... l, m, n
disintegration constant $[N=N_0\epsilon^{-\lambda t}]$ ... λ
dispersion $[=dn/d\lambda]$ ... σ
dispersion, angular $[=d\theta/d\lambda]$ ... D
dispersive power $[=(n_F-n_C)/(n_D-1)]$ ... ω
dispersive power, reciprocal of ... ν
displacement, angular ... θ
displacement components (of sound-bearing particle) ... ξ, η, ζ
displacement constant, Wien $[=\lambda_m T]$ ... b
displacement current ... $\dot{\mathbf{D}}$, $\dot{\mathbf{D}}/4\pi$
displacement, electric ... **D**
displacement flux density ... **D**
displacement flux, total electric $[=\int \mathbf{D}\cdot d\mathbf{A}]$ ... Ψ
displacement in general ... s
displacement, linear ... **s**
displacement, longitudinal (*acous.*) ... ξ
displacement of sound bearing particle ... ξ, η, ζ
displacement, transverse (*acous.*) ... η, (ξ, y, w)
displacement, volume (*acous.*) ... X
dissociation, degree of ... α
dissociation energy, nuclear ... Λ
distance, angular ... θ
distance between adjacent principal foci of two lens units ... Δ
distance between a principal plane of a lens system and the appropriate principal plane of a unit ... p
distance between corresponding points of grating (grating space) ... d
distance between image and principal focus of image space ... x'
distance between image and corresponding principal plane (image distance) ... s'
distance between lens units in an optical system ... d
distance from neutral axis to extreme fiber (*beams*) ... c
distance between object and principal focus of object space ... x
distance between object and corresponding principal plane (object distance) ... s
distance, image (*optics*) ... s'
distance, linear ... s
distance, object (*optics*) ... s
distance of approach, closest possible (in Rutherford scattering formula) ... b
distance, radial ... r
distribution function ... f, F
double layer, strength of surface ... P_s

effective (rms) current ... I
effective (rms) potential difference ... V
efficiency ... η
efficiency, luminous $[=F/P, \lambda=5550A]$ ... K
efficiency, monochromatic luminous, or luminosity, or visibility $[=F_\lambda/P_\lambda]$ ... K_λ
efficiency, voltage $[=V_{theor}/V_{actual}]$ (*electrolysis*) ... η_V
elastance, or reciprocal capacitance ... S
elasticity, modulus of shear ... G, n
elasticity, volume modulus of (reciprocal of compressibility factor) ... B
electric charge ... q, Q
electric charge density, volume ... ρ
electric conductance ... G, g
electric conductivity ... σ, (γ)

electric current density .... $\mathbf{J}$
electric displacement .... $\mathbf{D}$
electric displacement, total flux of [$=\int \mathbf{D}\cdot d\mathbf{A}$] .... Ψ
electric field strength .... $\mathbf{E}$
electric flux density [$\Psi=\int \mathbf{D}\cdot d\mathbf{A}$] .... $\mathbf{D}$
electric flux, total [$=\int \mathbf{D}\cdot d\mathbf{A}$] .... Ψ
electric moment of atom, dipole molecule, or dipole. μ, μ_e
electric moment, total .... $\mathbf{p}$
electric or mechanical equivalent of heat (Joule constant) .... J
electric polarization .... $\mathbf{P}$
electric potential, or potential difference V
electric quadrupole moment .... Q
electric resistance .... R, (r)
electric susceptibility [$\mathbf{P}=\eta\mathbf{E}$] .... η
electrochemical equivalent .... z
electromagnetic scalar potential .... φ
electromotive force, or emf .... $\mathcal{E}$, (E)
electron affinity [$=w/e$, volts] .... φ
electron affinity, gross [$=w_0/e$, volts] .... φ_g
electron emission, total (saturation) .... I_s
electronic charge .... $-e$
electronic mass (use m_e when m is magnetic quantum number) .... m
elongation, total (deflection) .... δ
emissive power, monochromatic .... J_λ, J_ν
emissive power, total .... J
emissivity .... ϵ'
  monochromatic (color, spectral) .... ϵ_λ'
  total value .... ϵ', ϵ_t'
emittance .... ϵ
end correction for a tube (*acous.*) .... α
energy
  density of radiant [$=dU/dV$] .... u
  internal or intrinsic (*thermodyn.*)
    per atom or molecule .... u, u_m
    per mole .... u, U, U_M
    per unit mass .... u
    total value .... U
  kinetic .... $E_k(T)$
    average molecular [$=\frac{1}{2}m\bar{\bar{v}}^2$] .... ϵ
  nuclear dissociation .... Λ
  of vibration .... E_v
  potential .... E_p, U, (V)
  radiant .... U
  self [$=mc^2$] .... ϵ
  spectral radiant [$=dU/d\lambda$] .... U_λ
  storage factor .... Q
  total .... E, U
  work .... W, (w)
enthalpy (heat content)
  per atom or molecule .... h, h_m
  per mole .... h, H, H_M
  per unit mass .... h
  total value .... H
entropy
  per atom or molecule .... s, s_m
  per mole .... s, S, S_M
  per unit mass .... s
  total value .... S
epoch angle .... ϵ, (φ)
equilibrium constant (products/reactants) .... K
equivalent conductivity (mho/mole) .... Λ
equivalent weight (*chem.*) .... M/z
equivalents, number of .... J
excitation potential .... V_e
expansion, total thermal .... δ
expansitivity, linear, or coefficient of linear expansion .. α
expansivity, volume, or coefficient of volume expansion. β
exposure [$=Et$] .... Σ
extinction coefficient [$=(1/cl)\log_{10}P_0/P$] .... ϵ

factor of safety .... N
Faraday constant or equivalent .... F
field strength, electric .... $\mathbf{E}$
field strength, or field intensity, magnetic .... $\mathbf{H}$
figure of merit of inductance [$=X/R$] .... Q
filament (subscript designation) .... f
fine-structure, or Sommerfeld constant [$=2\pi e^2/ch$] .... α
fluidity (reciprocal of viscosity) .... φ
flux density, electric .... $\mathbf{D}$
flux
  electric [$=\int \mathbf{D}\cdot d\mathbf{A}$] .... Ψ
  luminous .... F
  magnetic [$=\int \mathbf{B}\cdot d\mathbf{A}$] .... Φ
radiant, or radiant power [$=dU/dt$] .... Φ, P
flux density, magnetic .... $\mathbf{B}$
flux density, propagation .... Ξ
flux density, radiant, or irradiance [$=d\Phi/dA$ or dP/dA] .... W
  (When referring to a source is called radiancy, symbol $\mathcal{R}$)
focal length of image space .... f'
focal length of object space .... f
force .... F
force constant [$=-F/s$] .... k
force, gravitational .... F, F_g
force in a string or membrane .... F, (T)
force, shearing (in beam section) V
force, stretching ("tension"), in string or membrane ... F
formality .... F
free path .... l, λ
free path, mean .... $\bar{l}$, $\bar{\lambda}$
free path, Tait .... l_T, λ_T
frequency (*elec. circuits, mech., acous.*) .... f
frequency (*optics, quantum theory*) .... ν
frequency, angular [$=2\pi f$, radians/time] .... ω
frequency deviation [$=\Delta f/f_r$] .... δ
frequency of molecular collision .... Z
frequency, resonant .... f_r
frequency, rotational [rev/time] .... n
friction, coefficient of .... μ
  rolling .... μ_r
  sliding, or kinetic .... μ, (μ_k)
  starting, or static .... μ_s
  (Use f when μ designates the Poisson ratio)
fugacity .... f
function, distribution .... f, F

gain of amplifier, power .... A, A_p

gain of amplifier, voltage .... A, A_v
galvanometer deflection .... δ
gas amplification factor (*photoelectric tubes*) .... μ
gas constant, molecular, or Boltzmann constant .... k
gas constant
  universal [$pV=nRT=mRT/M$; $pv_M=RT$] .... R
  specific [$pv=rT$ where $v=V/m$ and $r=R/M$] .... r
generalized coordinate .... q
generalized momentum .... p
Gibbs function equation, integration constant of I
Gibbs function
  partial molal, or chemical, potential [$=\partial G/\partial n$] .... μ
  per atom or molecule .... g, g_m
  per mole [$=G/n$] .... g, G, G_M
  per unit mass .... g
  total value [$=H-TS$] .... G
grating lines, total number of .... N
grating space (distance between corresponding points) .... d
gravitational constant, Newtonian .... G
gravitational force .... F, F_g
grid control ratio (*thyratrons*) .... μ
grid or input admittance with plate load [$=g_g-jb_g$] .... y_g
grid, control. Use subscript $_g$, or $_{g1}$, etc., numbered from cathode
grid, injector. Use subscript $_{g4}$, etc., numbered from cathode
grid-plate transadmittance [$=g_{gp}-jb_{gp}$] .... y_{gp}
grid-plate transadmittance, real part of, or grid-plate transconductance .... g_{gp}, (g_m)
gyration, radius of .... k
gyromagnetic ratio .... g

half angle subtended at point object by objective of microscope .... α
half-life (*radioactivity*) .... T
Hamiltonian function or operator .... H
Hamiltonian function or operator, perturbing .... $\mathcal{H}$
Hamilton-Jacobi equation variable, time dependent .... W; (V)
head (*hydrodyn.*) .... h
heat capacity
  per atom or molecule .... c, c_m
  per mole .... c, C, C_M
  per unit mass (specific heat capacity) c
  total value C
  (Use subscript $_p$ or $_v$ to indicate constancy of pressure or of volume)
heat content, relative L
heat, electric or mechanical equivalent of (Joule equivalent) .... J
heat entering system .... q
heat flow path, length of .... L, l
heat of reaction or phase change (evaporation, fusion, sublimation, etc.)
  per atom or molecule .... $\Delta h, \Delta h_m$
  per mole .... $\Delta h, \Delta H, \Delta H_M$
  per unit mass .... Δh
  total value .... ΔH
heat, quantity of .... Q
heater, reference to (attach subscript to symbol) .... $_h$
height, depth and thickness .... h, y

Helmholtz function, or maximum isothermal work function
  per atom or molecule .... a, a_m
  per mole .... a, A, A_M
  per unit mass .... a
  total value [$=U-TS$] .... A
Henry law constant .... H
Hertzian vector .... Π
humidity, absolute .... ρ
humidity, relative .... r, f
hydrolysis constant .... K_h
hydrolysis, degree of .... h

ice point .... T_0, t_0
illuminance, or amount of illumination [$=dF/dA$] .... E
image distance .... s'
image length or height .... y'
imaginary unit ($=\sqrt{-1}$, same as right-angle turning operator; use j in electric circuits) .... j, i
impact parameter (*Rutherford scattering formula*) .... p
impedance; acoustic impedance .... Z
impedance, specific acoustic .... z
increment, finite .... δ, Δ
indices, Miller .... h, k, l
indices of Bragg reflection .... $\begin{cases} h_1[=nh] \\ h_2[=nk] \\ h_3[=nl] \end{cases}$
inductance L
inductance, mutual .... M, L_{12}, etc.
inductance, reciprocal .... Γ
induction coefficient, or partial capacitance coefficient c
induction density, electric .... $\mathbf{D}$
induction density, magnetic .... $\mathbf{B}$
induction, electric flux of [$=\int \mathbf{D}\cdot d\mathbf{A}$] .... Ψ
induction, magnetic flux of [$=\int \mathbf{B}\cdot d\mathbf{A}$] .... Φ
inductive reactance .... X_L
inductivity, or absolute permeability .... μ
inertia of photographic plate i
integration constant of Gibbs function equation I
intensity, acoustic (e.g., erg cm^{-2} sec^{-1}) I
intensity, luminous [$=dF/d\omega$] I
intensity of magnetization, or magnetization .... $\mathbf{M}$
intensity, radiant [$=d\Phi/d\omega=dP/d\omega$] .... J
interplanar distance (*Bragg law*) .... d
ionization, degree of (*solutions*) .... α
ionization potential .... V_i
irradiance, or radiant flux density [$=d\Phi/dA$ or dP/dA] .... $\mathcal{E}, H$

Joule equivalent (electric or mechanical equivalent of heat) .... J
Joule-Thomson (Kelvin) coefficient [$=\partial T/\partial p_H$] $\times(\partial T/\partial p)_H$ .... μ

Kelvin temperature .... T, (Θ)
Kerr constant .... K
kinetic energy .... E_k, (T)
kinetic energy, average molecular [$=\frac{1}{2}m\overline{v^2}$] .... ϵ
kinetic potential, or Lagrangian function L
$1/kT$ .... β

Lagrange function, or Lagrangian [$=E_k-E_p$] L
Landé factor .... g
leakage coefficient, magnetic .... σ
length l
length of arc or path .... s
length of image .... y'
length of object (*optics*) .... y
length of prism base .... t
length of vibrating string, rod or tube l
length, optical .... Δ
length, rest .... l_0
lens zone, radius of .... h
light, quantity of .... Q
light speed in vacuum c
linear acceleration .... $\mathbf{a}$
linear density [$=m/l$] .... λ, (ρ)
linear displacement .... $\mathbf{s}$
linear distance .... s
linear expansivity, or coefficient of linear expansion .... α
linear velocity .... $\mathbf{u}$, $\mathbf{v}$
lines of a grating, total number of .... N
load per unit displacement .... k
Lorentz unit [$=Be/4\pi mc^2$] L
Loschmidt number .... n_0
(number of molecules per unit volume at 0°C and 1 atm)
luminance, or brightness .... B
luminosity (luminous efficiency, monochromatic or visibility factor) [$=F_\lambda/P_\lambda=F_\lambda/\Phi_\lambda$] .... K_λ
luminous efficiency [$=F/P$, $\lambda=5550A$] .... K
luminous energy [$=\int F dt$] .... Q
luminous flux .... F

magnetic field strength, or magnetic field intensity .... $\mathbf{H}$
magnetic flux [$=\int \mathbf{B}\cdot d\mathbf{A}$] .... Φ
magnetic flux density, or magnetic induction .... $\mathbf{B}$
magnetic leakage coefficient .... σ
magnetic moment .... $\mathbf{m}$
magnetic moment of atom, molecule or dipole .... μ_m, μ
magnetic permeability .... μ
magnetic polarization, or magnetization .... $\mathbf{M}$
magnetic pole strength .... p, (m)
magnetic potential, scalar .... $\mathfrak{F}$
magnetic potential, vector .... $\mathbf{A}$
magnetic rotation, specific, or Verdet constant .... V, ω
magnetic shell strength I
magnetic susceptibility .... k
magnetic susceptibility, specific [$=k/\rho$] .... χ
magnetization [$=\mathbf{m}/V$] .... $\mathbf{M}$
magnetization, specific [$=M/\rho$] .... σ
magnetomotive force, or magnetic scalar potential .... $\mathfrak{F}$
magneton, orbital or Bohr .... μ_0
magneton, nuclear [$=\mu_0/1838$] .... μ_I
magnification, linear .... m
magnification, angular [$=\tan u'/\tan u$] .... γ
mass (general use) .... m
mass of atom or molecule .... m, m_m
mass of electron (use m_e when m is magnetic quantum number) .... m
mass per unit area .... σ, (ρ)
mass per unit length .... λ, (ρ)
mass per unit volume (density) .... ρ, (D)
mass, reduced $\left[\frac{1}{\mu}=\frac{1}{m_1}+\frac{1}{m_2}\right]$ .... μ
mass, rest .... m_0
mean free path .... $\bar{l}$, $\bar{\lambda}$
mean-life (*radioactivity*) .... τ
mechanical equivalent of heat, or Joule equivalent .... J
Miller indices .... h, k, l
mixing ratio, or water-vapor content .... w
modulation factor .... m
modulus of decay [$s=A\cos(\omega t+\varphi)\exp(-t/\tau)$] .... τ
modulus of elasticity, shear .... n
modulus of elasticity, volume (reciprocal of compressibility factor) .... B
modulus of elasticity, Young .... E, Y
modulus of section [$=I/c$, where c is distance of neutral axis from extreme fiber] .... Z
molecular collision frequency [=no./time] .... Z
molecular concentration [$=N/V$] .... n
at 0°C and 1 atm (Loschmidt number) .... n_0
molecular conductivity .... μ
molecular cross section, effective .... σ, S
molecular density, or Loschmidt number .... n_0
molecular diameter (collision) .... σ
molecular gas constant, or Boltzmann constant .... k
molecular mass .... m, $\overline{m}$
molecular kinetic energy, average [$=\frac{1}{2}m\bar{v}^2$] .... ϵ
molecular volume V
molecular weight .... M
molecules, number of .... N
mole factor, or van't Hoff coefficient i
moles, number of .... n
moment, electric .... $\mathbf{p}$
moment, magnetic .... $\mathbf{m}$
moment of area [$=\int y\,dA$] .... Q
moment of atom, molecule or dipole, electric .... μ, μ_e
moment of atom, molecule or dipole, magnetic .... μ_m, μ
moment of inertia .... I, I_{xx}, etc., (J)
moment of inertia, areal I
polar [$=\int r^2 dA$] .... J
rectangular [$=\int y^2 dA$]
momentum, generalized .... p
mutual inductance .... M, L_{12}, etc.

Naperian base .... ϵ, e
natural or resonant frequency .... f_r
normality C
normalization factor (*probability*) .... N
nuclear dissociation energy .... Λ
nuclear magneton [$=\mu_0/1838$] .... μ_I
nuclear radius .... r
number .... n
number of atoms or nuclei (if more than one type is considered) .... $N_{1,2}\ldots$
number of atoms or nuclei at time t (*radioactivity*) .... N
number of atoms or nuclei, initial (*radioactivity*) .... N_0
number of atoms or nuclei, initial (if more than one type is considered) .... $N^\circ_{1,2}\ldots$

number of components (Gibbs phase rule) ... n
number of equivalents ... J
number of lines of a grating, total ... N
number of molecular collisions per unit time, or collision frequency ... Z
number of molecules (total) ... N
number of molecules per mole at 0°C and 1 atm (Avogadro number) ... N_0
number of molecules per unit volume (molecular concentration) ... n
at 0°C and 1 atm (Loschmidt number) ... n_0
number of moles ... n
number of phases (*elec. circuits*) ... m
number of revolutions or rotations per unit time ... n
number of turns or of conductors ... N

object distance (*optics*) ... s
object length or height (*optics*) ... y
optical attenuation ... D
optical density [$=\log_{10}P_0/P$] ... D
optical tube length ... Δ
optical path difference ... Δ
optical transmittance or transmission factor [$=P/P_0$] ... τ
orbital, or Bohr, magneton ... μ_0
order of spectrum ... m, (n)
oscillation period ... T
osmotic pressure ... p, Π, (π)

path difference, optical ... Δ
peak current ... $\hat{I}$, I_p, (I_{pk})
peak potential ... $\hat{V}$, V_p, V_{pk}
Peltier coefficient ... Π
Peltier potential ... V_π
periodicity [$=2\pi f$] ... ω
periodicity, resonant ... ω_r
period of a periodic motion ... T
permeability or inductivity, magnetic ... μ
permeability, reciprocal, or reluctivity ... ν
of free space ... μ_0
permeability, relative magnetic [$=\mu/\mu_0$ mks units] ... μ_r
permeance [$=L/N^2$] ... Λ
permittance, or capacitance ... C
permittivity (mks units; *see* capacitivity) ... ϵ
of free space ... ϵ_0
relative [$=\epsilon/\epsilon_0$] ... ϵ_r
phase angle ... φ
phase constant [$\gamma=\alpha+j\beta$] ... β
photoelectric threshold frequency ... ν_0, (f_0)
photographic plate, inertia of ... i
phototube, sensitivity of
static ... S
dynamic ... s
piezoelectric strain constant or modulus ... δ
piezoelectric stress constant ... ϵ
Planck constant ... h
Planck function [$=-A/T$] ... Ψ
Planck radiation law constants

$$\left[\frac{J_\lambda}{A\cos\theta}=N_\lambda=c_1/\lambda^5\left(\exp\frac{c_2}{\lambda T}-1\right)\right] \quad \ldots \quad c_1, c_2$$

plane spacing in a crystal, Bragg ... d
plate power ... P_p
plate resistance ... r_p
Poisson ratio ... μ, ν
polar coordinates ... r, θ
polarization, electric ... $\mathbf{P}$
polarization, surface, or strength of double layer (*elec.*) ... $\mathbf{P}_s$
polarization, magnetic, or magnetization ... $\mathbf{M}$
pole strength, magnetic ... p, (m)
position vector ... $\mathbf{r}$
potential coefficient, partial ... p
potential difference, average ... V_{av}, $\bar{V}$
potential difference, contact or Volta ... V_v
potential difference, excitation ... V_e
potential difference, instantaneous ... v
potential difference, maximum ... V_m, (V_{max})
potential difference, maximum peak ... $\hat{V}_m$, V_{mp}
potential difference, peak ... $\hat{V}$, V_p, (V_{pk})
potential difference, Peltier ... V_π
potential difference, quiescent ... $\bar{V}$
potential difference, rms or effective ... V
potential difference, Seebeck ... V_s
potential difference, Thomson ... V_t
potential difference, steady d-c ... V
potential, electric ... V
potential, electromagnetic scalar ... φ
potential energy ... E_p, U, (V)
potential, inner (*metals*) ... V, V_i
potential, ionization ... V_i
potential, kinetic, or Lagrange function ... L
potential, magnetic scalar (magnetomotive force) ... $\mathfrak{F}$
potential, magnetic vector ... $\mathbf{A}$
potential, velocity (*hydrodyn.*) ... φ
power ... P
(Use same system of subscripts as for current except that P, unmodified, signifies average power)
power, active (*elec.*) ... P
power, input ... P_i
power of lens system, or refracting power [$=n/f$, diopters] ... D
power, output ... P_o
power, plate ... P_p
power, radiant, or radiant flux ... P, Φ
power, thermoelectric ... Q
Poynting vector ... Π
pressure [$=F/A$] (Use P for total pressure when p represents vapor pressure) ... p
amplitude of simple harmonic ... P
static ... P_0
varying ... p
pressure, critical ... p_c
pressure, osmotic ... p, Π, (π)
pressure, partial ... p
pressure, vapor ... p
probability ... P
product of inertia ... I_{xy}, etc.
propagation constant [$=\alpha+j\beta$] ... γ
propagation flux density ... Ξ
pulsatance, or periodicity [$=2\pi f$] ... ω

Q-factor, or quality factor of reactor [$=X/R$] ... Q

quadrupole moment, electric ... Q
quantity of electric charge ... q, Q
quantity of heat ... Q
quantity of light, or luminous energy [$=\int F dt$] ... Q
quantum number
  azimuthal or orbital ... l
  azimuthal or orbital, total ... L
  hyperfine [$=I+J$] ... F
  inner ... j
  inner, total ... J
  magnetic ... m
  magnetic, total ... M
  principal ... n
  rotational ... R
  spin ... s
  spin, total ... S
  spin, nuclear ... I
  vibrational ... v
    (Strictly, spin is not a quantum number)

radial distance ... r
radiancy, or radiance, or radiant flux density [$=d\Phi/dA=dP/dA$] ... $\mathcal{R}, W$
radiant energy ... U
radiant energy density [$=dU/dV$] ... u
radiant energy, spectral (monochromatic) [$=dU/d\lambda$] ... U_λ
radiant flux density [$=d\Phi/dA$ or dP/dA] ... W
  (When referring to a source is called radiancy, symbol $\mathcal{R}$)
radiant intensity [$=d\Phi/d\omega=dP/d\omega$] ... J
radiant intensity, spectral [$=dJ/d\lambda$] ... J_λ
radiant power, or radiant flux [$=dU/dt$] ... Φ, P
radiation law constants (*see* Planck radiation law constants) ... c_1, c_2
  Stefan-Boltzmann ... σ
  Wien ... b
radius ... r
radius, Bohr [$=h^2/4\pi^2me^2$] ... a_1
radius of circle of least confusion ... Z
radius of curvature ... ρ
radius of gyration ... k
radius of image of a point ... Z
radius of lens zone ... h
radius of nucleus ... r
radius of tube, disk or membrane (*acous.*) ... $a, (R)$
radius vector ... $\mathbf{r}$
range (*radioactivity*) ... R
ratio of speed to speed of light [$=v/c$] ... β
reactance, acoustic reactance ... X
  capacitive ... X_C
  inductive ... X_L
reactance, specific acoustic [XA] ... x
reaction rate, specific, or reaction velocity constant ... k
reaction velocity [$=-dc/dt$] ... u
reciprocal capacitance, or elastance ... S
reciprocal impedance, or admittance ... Y
reciprocal inductance ... Γ
reciprocal of dispersive power ... ν
reciprocal permeability, or reluctivity ... ν
recombination, coefficient of ... α
rectangular coordinates ... x, y, z
reflectance, or reflection factor [$=P/P_0$] ... ρ
reflectivity ... ρ'
refractive index ... $n, (\mu)$
refractive index, group ... $n_g, (\mu_g)$
relative dielectric coefficient, or specific inductive capacity ... ϵ, ϵ_r
relative magnetic permeability ... μ, μ_r
relativity ratio [$=v/c$] ... β
reluctance ... $\mathcal{R}$
reluctivity [$=1/\mu$] ... ν
resilience, coefficient of, or coefficient of restitution ... e
resistance, acoustic ... R
resistance, electric ... $R, (r)$
resistance, plate ... r_p
resistance, radiation ... R
resistance, specific, or resistivity ... ρ
resistance, specific acoustic [$=RA$] ... r
resistance, thermal ... R
resistance, thermal coefficient of ... α
resistivity, or specific resistance ... ρ
resistivity, thermal ... $1/k$
resolving power of telescope, angular ... α
resonance frequency ... f_r
resonance periodicity ... ω_r
restitution, coefficient of, or resilience ... e
restoring force per unit displacement ... k
reverberation time ... T
revolutions or rotations per unit time ... n
Richardson equation factors [$I_s=AT^2e^{-b/T}$] ... A, b
right-angle turning operator ($=\sqrt{-1}$; use j in elec. circuits) ... j, i
rigidity, or shear modulus of elasticity ... n
rms (effective) potential difference ... V
  current ... I
rotational frequency [rev/time] ... n
rotation, angle of optical ... θ
rotation, specific [$=\theta/lc$] ... α
rotation, specific magnetic, or Verdet constant ... V, ω
Rydberg constant ... R
Rydberg constant for infinite mass ... R_∞

safety factor ... N
sag or deflection of beam ... δ
scattering coefficient (turbidity) ... s
self energy [$=mc^2$] ... ϵ
sensitivity of phototube
  static ... S
  dynamic ... s
shearing force in beam section ... V
shear modulus of elasticity ... G, n
shell, strength of magnetic ... I
slip (*elec. machinery*) ... σ, s
slit width (transparent portion) ... a
Sommerfeld, or fine-structure, constant [$=2\pi e^2/ch$] ... α
spacing of Bragg planes in a crystal ... d
specific heats, ratio of [$=c_p/c_V$] ... γ
specific inductive capacity (*see* capacitivity)
specific magnetic rotation, or Verdet constant ... V, ω
specific resistance, or resistivity ... ρ

specific volume [$=V/m$] v
specific weight [w/V] .... γ
spectral radiant energy [$=\mathrm{d}U/\mathrm{d}\lambda$] .... U_λ
speed .... u, v
speed at time t .... u, u_t, v, v_t
speed, angular .... ω
speed, average .... $\bar{u}, \bar{v}, u_{av}, v_{av}$
speed, initial .... u_0, v_0
speed, linear or particle .... u, v
speed, most probable .... $\hat{u}, \hat{v}, (c_0), \alpha$
speed of light in vacuum; speed of sound c
speed, rms .... $\bar{\bar{u}}, \bar{\bar{v}}, (C)$
spin .... s
  nuclear I
  total .... S
spring constant [$=-F/s$] .... k
square-root of minus one [$=\sqrt{-1}$] .... j, i
  (This is the special case, where $\varphi=\pi/2$, of the operator $\exp j\varphi$ which turns a vector through the angle φ; use j in elec.)
statistical weight or degeneracy .... g
Stefan-Boltzmann constant [$=\mathcal{R}/T_4$] .... σ
steradiancy [$=\mathrm{d}J/\mathrm{d}A \cos\theta$] .... $\mathcal{B}, N$
strength of magnetic pole .... $p, (m)$
strength of magnetic shell I
summation operator .... Σ
super-compressibility factor .... y
surface tension .... γ, σ
susceptance .... B, b
susceptibility, electric [$\mathbf{P}=\eta\mathbf{E}$] .... η
susceptibility, volume magnetic [$\mathbf{M}=k\mathbf{H}$] .... k
susceptibility, specific magnetic [$=k/\rho$] .... χ

Tait free path .... l_T, λ_T
temperature, absolute .... $T, (\Theta)$
temperature coefficient of resistance .... α
temperature, critical .... t_c, T_c
temperature, Debye characteristic .... Θ
temperature, dew point .... τ
temperature of ice point
  absolute .... T_0
  ordinary .... t_0
temperature, ordinary .... $t, (\theta)$
tension (force) in spring or membrane .... F
thermal conductivity .... k
thermal expansion, coefficient of, or expansivity
  linear .... α
  volume [$=\alpha_1+\alpha_2+\alpha_3$] .... β
thermal expansion, total .... δ
thermal resistance .... R
thermal resistivity .... $1/k$
thermoelectric power .... Q
thickness and height .... h, y
Thomson coefficient .... σ
time (Use τ when t is used for temperature) .... $t, (\tau)$
time constant .... τ
time-dependent variable in Hamilton-Jacobi equation .... $W, (V)$
time dependent wave function .... ψ
time independent wave function .... u
time, reverberation .... T
torque per unit twist, or torsion constant .... k
transadmittance, grid-plate [$=g_{pg}-jb_{pg}$] .... y_{pg}
transconductance, grid-plate (real part of grid-plate transadmittance) .... $g_{pg}, (g_m)$
transconductance at zero frequency and constant plate potential [$=(\partial i_p/\partial v_g)_{v_p}$] .... s_p
transmittance [$=P/P_0$] .... τ
turbidity (scattering coefficient) .... s
turning operator, 90°($=\sqrt{-1}$; use j in elec. circuits) .... j, i

u/c .... β

valence .... z
Van der Waals constants $\left[\left(p+\frac{n^2a}{V^2}\right)(V-nb)=nRT\right]$. a, b
Van't Hoff coefficient i
vapor density .... ρ
vapor pressure .... p
vapor pressure constant i
variable .... x, y, z
variable in Hamilton-Jacobi equation, time-dependent .... $W, (V)$
vector in X-direction, unit .... $\mathbf{i}$
  in Y-direction, unit .... $\mathbf{j}$
  in Z-direction, unit .... $\mathbf{k}$
  tangent to path, unit .... $\boldsymbol{\tau}$
vector, position .... $\mathbf{r}$
vector turning operator [$=\sqrt{-1}$] .... j, i
  (This is the special case, where $\varphi=\pi/2$, of the operator $\exp j\varphi$ which turns a vector through the angle φ; use j in elec. circuits)
velocity .... $\mathbf{u}, \mathbf{v}$
velocity, angular .... $\boldsymbol{\omega}$
velocity, average .... $\bar{\mathbf{u}}, \bar{\mathbf{v}}, \mathbf{u}_{av}, \mathbf{v}_{av}$
velocity at time t .... $\mathbf{u}, \mathbf{v}, \mathbf{u}_t, \mathbf{v}_t$
velocity, components of linear or particle
  $\mathbf{u}_x, \mathbf{u}_y, \mathbf{u}_z$ ($\mathbf{u}, \mathbf{v}, \mathbf{w}$)
velocity constant, reaction .... k
velocity, group .... $\mathbf{u}, \mathbf{v}, \mathbf{u}_g, \mathbf{v}_g$
velocity, initial .... $\mathbf{u}_0, \mathbf{v}_0$
velocity, ionic (with suitable subscript) .... u
velocity, linear or particle .... $\mathbf{u}, \mathbf{v}$
velocity, most probable .... $\hat{\mathbf{u}}, \hat{\mathbf{v}}, (\mathbf{c}_0, \alpha)$
velocity of light in vacuum c
velocity of sound or other waves .... c, v
velocity, phase or wave .... $u_w, u_\varphi, (w)$
velocity potential .... φ
velocity, reaction [$=-\mathrm{d}c/\mathrm{d}t$] .... u
velocity, rectangular components .... $\mathbf{u}_x, \mathbf{u}_y, \mathbf{u}_z, \mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z$
velocity, rms .... $\bar{\bar{u}}, \bar{\bar{v}}, (C)$
velocity, specific reaction, or reaction velocity constant .... k
Verdet constant .... V, ω
viscosity, coefficient of kinematic [$=\eta/\rho$] .... ν
viscosity, coefficient of .... η
visibility factor, or luminosity, or monochromatic luminous efficiency [$=F_\lambda/P_\lambda=F_\lambda/\Phi_\lambda$] .... K_λ
voltage (*see* potential difference) V
Volta potential, or contact potential .... V_V

volume
- critical .... V_c
- of a cavity or room V
- of configuration-space V
- of phase space .... Ω
- per atom or molecule .... v, v_m
- per mole .... v, V, V_M
- per unit mass (specific volume) v
- total .... V, (τ)

volume displacement (*acous.*) X
volume electric charge density .... ρ
volume expansivity, or coefficient of volume expansion .. β
volume modulus of elasticity (reciprocal of compressibility factor) .... B
volume susceptibility, magnetic $[\mathbf{M}=k\mathbf{H}]$ .... k

water-vapor content, or mixing ratio .... w
wave function, time dependent .... ψ
wave function, time independent .... u
wavelength .... λ
wavelength constant $[=2\pi/\lambda]$ .... k
wavelength, effective .... λ_e
wave number .... σ
weight $[=mg]$ .... w, F, (W)
weight, equivalent (*chem.*) .... M/z
weight, statistical, or degeneracy .... g
weightivity, or specific weight $[w/V]$ .... γ
width or breadth .... b
width of slit (transparent portion) .... a
Wien displacement constant $[=\lambda_m T]$ .... b
work .... W, (w)
work function, gross .... w_g
work function per unit charge, gross, or gross electron affinity $[d\varphi_g=dw_g/de$, volts$]$ .... φ_g
work function, net .... w
work function per unit charge, net, or electron affinity $[d\varphi=dw/de$ volts$]$ .... φ

Young modulus of elasticity .... E, Y

zone of lens, radius of .... h

SUBSCRIPTS

adiabatic: attach subscript $_s$ to symbol
adsorbed: attach subscript $_a$ to symbol
average value: use subscript $_{av}$, or bar over symbol (e.g., I_{av}, $\bar{I}$)
blackbody, reference to: attach subscript $_b$ to symbol
cathode: attach subscript $_c$ to symbol
control grid: subscript $_g$
(For several grids, use subscripts $_{g1}$, $_{g2}$, ..., beginning at cathode)
critical properties: subscript $_c$
gas or vapor, reference to: attach subscript $_g$ to symbol
grid, screen: use subscript $_s$ or $_{g2}$, etc., numbered from cathode
grid, suppressor: use subscript $_{su}$ or $_{g3}$, etc., numbered from cathode
liquid, reference to: attach subscript $_L$ to symbol
maximum: attach subscript $_m$ or $_{max}$ to symbol
minimum: attach subscript $_{min}$ to symbol
monochromatic: attach subscript $_\lambda$ to symbol
normal (perpendicular), reference to: attach subscript $_n$ to symbol
perpendicular (normal), reference to: attach subscript $_n$ to symbol
plate, reference to: attach subscript $_p$ to symbol
reduced properties, (fluids): attach subscript $_r$ to symbol
solid, reference to: attach subscript $_X$ to symbol (*x*tal)
source, reference to: attach subscript $_s$ to symbol
spectral or monochromatic: attach subscript $_\lambda$ to symbol
transition between polymorphic forms, reference to: attach subscript $_t$ to symbol
vapor or gas, reference to: attach subscript $_G$ to symbol

Supplementary List

Listed in this category are those magnitudes which the Committee feels should be assigned standard letter symbols but upon which agreement has not been reached.

absorptance
absorptivity
acoustic capacitance
acoustic compliance
acoustic mass, or inertance
amplitude of velocity (*acous.*)
coupling coefficient (*elec. circuits*)
dilatation
dissipation function
elongation, unit
energy density (*acous.*)
energy flux (*acous.*)
energy reflection coefficient (*acous.*)
energy transmission coefficient (*acous.*)
impulse, linear and angular
intensity level (*acous.*)
loudness
loudness level
mechanical impedance (*acous.*)
mechanical reactance (*acous.*)
mechanical resistance (*acous.*)
mobility, ionic
moment of force, or torque
momentum, linear and angular
number of molecules per unit mass
power factor
specific gravity
strain components
strain, unit normal
strain, unit shearing
stress components
stress, total shearing
stress, unit normal
stress, unit shearing
volume current (*acous.*)

Other Arrangements of Symbols

There are three possible ways to arrange a list of symbols: (i) by name of the quantity, (ii) by symbol and (iii) by so-called "fields" or "branches." All three arrangements have their use, and at the start of the work we considered them carefully. To include all three in a report seemed desirable but out of the question because the resulting publication would be both bulky and expensive. In its present form, the ASA Standard already occupies 26 pages.

Fortunately for the Committee, the list arranged by symbol has been published in the *Handbook of Chemistry and Physics* for several years,[2] and our new list will be included in future editions.[9] This handbook enjoys a wide

distribution so that the problem of finding the name for a symbol is readily solved.

An arrangement by field would involve much duplication, for the same quantity frequently occurs in several fields. Engineers working in a restricted field may not be as conscious of this problem as teachers of elementary physics who, in the course of two semesters, cover the whole of physics. We have been asked for classifications not only of mechanics, heat, light, sound and electricity but also electronics, meteorology, crystal physics and a host of other fields. The task of preparing such lists seemed formidable and would have delayed the issuance of our report.

A feature of the present list is the careful cross-indexing. Practically every common term used in physics and engineering is listed. While finding an unknown term should be considered, this problem does not appeal to us as being either common or difficult. A person reading a text or paper is never so unfamiliar with the field that he has no ideas whatsoever about the meaning of a symbol. It generally can only mean one of two or three possibilities and this is settled quickly enough.

As an example of the cross-indexing, consider the symbol J in a discussion of the efficiency of heat engines: its meaning can be found under "heat," "Joule equivalent" and "mechanical equivalent." As another example, on pages 171 and 172 there are 26 entries formed from the root "elec;" this provides a good start toward a list of common electrical terms.

Acknowledgments

During its ten years of service, the Committee has corresponded with numerous members of the American Association of Physics Teachers and with others interested in standard symbols. With rare exceptions, we have found them to be understanding of the problems facing the Committee and to be helpful and encouraging. Specific acknowledgments were made in our previous report.[1]

The Chairman wishes to express to the other members of the Committee his personal appreciation of their fine cooperation. This has been shown repeatedly in their willingness to accept assignments and in their careful checking of important details. Their tolerant acceptance of the Chairman's decisions in certain controversial cases has made his task both pleasant and illuminating.

THE COMMITTEE ON LETTER SYMBOLS

HAROLD K. HUGHES, *Chairman, Socony-Vacuum Laboratories, Brooklyn 22, New York.*
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M. W. ZEMANSKY, *College of the City of New York*
Former Members of the Committee: GRANT O. GALE; MADELINE MITCHELL TATE.

The Republic *Does* Have Need of Savants!

The responsibility for forming ideals and fixing standards does not belong to statesmen alone. It belongs, and now perhaps more largely than ever before, to the intellectual leaders of the nation, and especially to those who address the people in the universities and through the press. Teachers, writers, journalists are forming the mind of modern nations to an extent previously unknown. Here they have opportunities such as have existed never before, nor in any other country, for trying to inspire the nation with a love of truth and honor, with a sense of the high obligations of citizenship, and especially of those who hold public office.—JAMES BRYCE.

Book-Length Biographies of Physicists and Astronomers—Addendum

Thomas James Higgins
Illinois Institute of Technology, Chicago 16, Illinois

In the decade 1935–1945 the author endeavored—as a satisfying and profitable avocation—to seek out, to read and to record all *book-length* biographies (individual and collected) in English of physicists and astronomers, mathematicians, chemists, and engineers, metallurgists and industrialists. Subsequently, the titles of pertinent items were published in a series of four bibliographies;[1–4] the titles were obtained by search of (i) the stacks and card catalogs of the important public, university and technical libraries located in the East and Middle West; (ii) the accumulated catalogs of the principal American and British publishers of technical and scientific books; (iii) the lists of offerings, over a decade, of the larger American and British dealers in used and rare technical and scientific works; (iv) much relevant miscellaneous bibliographical reference works: book review journals, printed catalogs of American and British private, public and national libraries, and kindred aids.

Subsequently, reprints of each of these bibliographies were sent to certain major libraries (in both America and Great Britain) which the author had not been able to visit in person, together with a request for the titles of additional items, if any, contained in the library. A limited number of titles of rather obscure items stemmed from these requests. Several others were contributed by interested American and British readers of the published bibliographies. These titles, together with those of recently published items, comprise a series of four short addenda to appear—it is hoped—in the periodicals that contained the corresponding bibliographies.

In consideration of the manner of compilation it is believed that practically all significant book-length biographies in English of physicists and astronomers are encompassed in the original bibliography or in the following addendum. In consequence of this definitive character, these listings are of obvious worth to all who are professionally interested in the history of physics and astronomy or—more broadly—in the history of science in general. In particular, they can be very useful in preparing the biographical content of a course of study utilizing the historical approach in the manner delineated by J. B. Conant in his recent well-received and widely discussed book, *On Understanding Science: An Historical Approach.*[5]

Physicists—Individual

L. Rosenfeld, *Niels Bohr, An essay dedicated to him on the occasion of his sixtieth birthday, October 7, 1945* (North Holland Publishing Co., Amsterdam, 1945).

J. F. Fulton, *A bibliography of the Honourable Robert Boyle, Fellow of the Royal Society* (Oxford Univ. Press, 1932). Contains very interesting biographical material.

A. C. Crehore, *Autobiography* (W. G. Berner, 1199 Lander Road, Gates Mills, Ohio, 1944).

J. Thomson, *An account of the life, lectures and writings of William Cullen* (Edinburgh, 1832; ed. 2, 2 vols., 1859).

Eleanor Doorly, *The radium woman* (William Heinemann, London, 1932). A youth edition of E. Curie, *The Life of Madame Curie.*

H. E. Armstrong, *James Dewar, 1842–1923* (E. Benn, London, 1924).

P. Frank, *Einstein, his life and times* (Knopf, 1947).

D. Marianoff and P. Wayne, *Einstein, an intimate study of a great man* (Doubleday, Doran, 1944).

A. Moszkowski, *Einstein, the searcher: his work explained from dialogues with Einstein,* tr. by H. L. Brose (Methuen and Co., London; E. B. Dutton, New York, 1921).

J. Parton, *Life and times of Benjamin Franklin* (Mason and Hamlin, Boston; Mason Brothers, New York, ed. 1, 1864).

The record of the celebration of the two hundredth anniversary of the birth of Benjamin Franklin, under the auspices of the American Philosophical Society, held at Philadelphia for promoting useful knowledge, April the seventeenth to April the twentieth, A. D. nineteen hundred and six (American Philosophical Society, Philadelphia, 1906).

Henry Gordon Gale 1874–1942, a record of the memorial services conducted by members of the faculty of the Uni-

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[1] T. J. Higgins, "Book-length biographies of physicists and astronomers," *Am. J. Physics* **12**, 31–39, 234–236 (1944). *Errata:* p. 31, for Bridges read Bridge; p. 33, for Randall read Randell; p. 34, for Grey read Gray.

[2] T. J. Higgins, "Biographies and collected works of mathematicians," *Am. Math. Mo.* **51**, 433–445 (1944).

[3] T. J. Higgins, "Book-length biographies of chemists," *Sch. Sci. and Math.* **44**, 650–665 (1944).

[4] T. J. Higgins, "Biographies of engineers, metallurgists and industrialists," *Bull. Bibliography* **18**, 207–210, 235–239 (1946); **19**, 10–12, 32 (1947).

[5] J. B. Conant, *On understanding science; an historical approach* (Yale Univ. Press, 1947).

versity of Chicago on November 18, 1942 at the Joseph Bond Chapel, Chicago, Illinois with some additional material from other sources (Privately printed, Chicago, 1943).

S. P. Thompson, *William Gilbert, and terrestrial magnetism in the time of Queen Elizabeth* (Charles Wittingham and Co., London, 1903).

A collection of papers in memory of Sir William Rowan Hamilton (Scripta Mathematica, Yeshiva College, New York, 1945).

A. W. Rücker, *Hermann von Helmholtz* (Cassell, London, 1895).

A. E. Bell, *Christian Huygens and the development of science in the seventeenth century* (Edward Arnold, London, 1948).

S. P. Thompson, *The Kelvin lecture, the life and work of Lord Kelvin* (E. and F. N. Spon, London; Spon and Chamberlain, New York, 1908).

H. H. Langton, *Sir John Cuningham McLennan, A memoir* (Oxford Univ. Press; Toronto Univ. Press, 1939).

J. Macalay, *James Clerk Maxwell* (New Biography series, No. 14, London, 1888).

W. W. R. Ball, *An essay on Newton's "Principia"* (Macmillan, 1893). Primarily biographical.

J. B. Biot, *Sir Isaac Newton*, tr. by H. C. Elphinstone (Society for The Diffusion of Useful Knowledge, Baldwin and Craddock, London, 1829).

J. Craig, *Newton at the mint* (Cambridge Univ. Press and Macmillan, 1946).

I. Hartill, *Recollections of Newton House* (J. Clarke, London, 1914).

H. McLachlan, *The religious opinions of Milton, Locke and Newton* (Manchester Univ. Press, Manchester, 1941).

M. H. Nicholson, *Newton demands the muse. Newton's Opticks and the eighteenth century prose* (Princeton Univ. Press, 1946). Not biographical, but most interesting.

H. W. Turnbull, *The mathematical discoveries of Newton* (Blackie and Sons, London, Glasgow and Bombay, 1945).

E. F. Northrup, *Zero to eighty, being my lifetime doings, reflections and inventions, also my journey around the moon, Akkad Pseudoman* (Scientific Publishing Co., Princeton, N. J., 1937). Amusing fictitious autobiography of a scientist born in 1920 writing in 2000.

E. Cailliet, *The clue to Pascal* (Wesminister Press, Philadelphia, 1943).

E. Cailliet, *Pascal: genius in the light of scripture* (Westminister Press, Philadelphia, 1945).

M. Duclaux, *Portrait of Pascal* (T. Fisher Unwin, London, 1927).

H. F. Stewart, *Blaise Pascal* (Oxford Univ. Press, 1942).

H. F. Stewart, *The holiness of Pascal* (Cambridge Univ. Press, 1915).

H. F. Stewart, *The secret of Pascal* (Cambridge Univ. Press, 1941).

W. Cobbett, *Observations on the emigration of Dr. Joseph Priestly and on the several addresses delivered to him on his arrival at New York* (Thomas Bradford, Philadelphia, ed. 3, 1895).

O. Glasser, *Dr. W. C. Röntgen* (C. C. Thomas, Springfield, Ill., 1945).

I. B. N. Evans, *Rutherford of Nelson* (Penguin Books, London, 1943).

E. M. Odling, *Memoir of the late Alfred Smee, F.R.S. by his daughter with a selection from his miscellaneous writings* (G. Bell and Sons, London, 1878).

A. S. Eve and C. H. Creasey, *Life and work of John Tyndall* (Macmillan, 1945).

H. Gurney, *Memoir of the life of Thomas Young, M.D., F.R.S. with a catalogue of his works and essays* (J. and A. Arch, London, 1831).

Physicists—Collected

E. T. Bell, *Men of Mathematics* (Simon and Schuster, 1937). Archimedes, Descartes, Pascal, Newton, the Bernoullis, Euler, Lagrange, Laplace, Fourier, Gauss, Hamilton, Poincaré, among others.

S. K. Bolton, *Famous men of science*, revised by E. W. Sanderson (Crowell, ed. 3, 1946). Copernicus, Galileo, Newton, Herschel, Faraday, Pasteur, Kelvin, Edison, the Curies, Marconi, the Comptons, Einstein and Urey, among others.

J. G. Crowther, *British scientists of the nineteenth century* (Penguin Books, London, vol. 1, 1940; vol. 2, 1941). Davy, Faraday, Joule, Kelvin, Maxwell.

J. G. Crowther, *Famous American men of science* (Harmondworth, London; Penguin Books, 1944). Vol. 1, Franklin and Henry; vol. 2, Edison and Gibbs.

J. C. Geikie, *Michael Faraday and Sir David Brewster, philosophers and Christians; lessons from their lives* (London, 1868).

W. Godwin, *Lives of the necromancers: or, an account of the most eminent persons in successive ages who have claimed for themselves, or to whom has been imputed by others, the exercise of magical powers* (J. F. Mason, London, 1834; Harper and Brothers, New York, 1835). Roger Bacon, Raymond Lully, Paracelsus, Cardan and Dee, among others.

R. J. Harvey-Gibson, *The master thinkers: vignettes in the history of science, with portraits* (T. Nelson and Sons, London, 1928).

S. H. Jenkinson, *New Zealanders and science* (Department of Internal Affairs, Wellington, 1940). Rutherford and Gifford, among others.

W. Nida and S. H. Nida, *Men of science and their inventions* (Harrap, London, 1927). Very elementary.

F. J. Oppenheimer, *Ezekiel to Einstein; Israel's gifts to science and invention* (Liveright Publishing Corp., New York, 1940). Primarily a listing, but most interesting because of the subject.

C. S. Slichter, *Science in a tavern; essays and discoveries in the making* (Univ. of Wisconsin Press, ed. 1, 1938; ed. 2, 1940). Kelvin, among others.

E. Slosson, *Major prophets of today* (Little, Brown, 1914). Poincaré, among others.

Astronomers—Individual

H. S. Jones, *John Couch Adams and the discovery of Neptune* (Cornell Univ. Press, 1947).

R. E. Berry, *Yankee stargazer: the life of Nathaniel Bowditch* (McGraw-Hill, 1941).

N. I. Bowditch, *Memoir of Nathaniel Bowditch by his son Nathaniel Ingersoll Bowditch* (Little and Brown, 1840; Cambridge Univ. Press, 1884).

J. Pickering, *Eulogy on Nathaniel Bowditch, LL.D., President of the American Academy of Arts and Sciences; including an analysis of his scientific publications* (Little and Brown, Boston, 1838).

A. Stanford, *Navigator; the story of Nathaniel Bowditch* (Morrow, 1927).

D. A. White, *An eulogy on the life and character of Nathaniel Bowditch* (Office of the Gazette, Salem, Mass., 1838).

A. Young, *A discourse on the life and character of the Hon. Nathaniel Bowditch, LL.D., F.R.S.* (Little and Brown, Boston, 1838).

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J. A. Gade, *The life and times of Tycho Brahe* (Princeton Univ. Press, 1947).

A. Armitage, *Sun, stand thou still: The life and work of Copernicus the astronomer* (Henry Schuman, New York, 1947).

H. Kesten, *Copernicus and his world*, tr. by E. B. Ashton and N. Guterman (Roy Publishers, New York, 1945).

Nicholas Copernicus: A tribute of nations, ed. by Stephen P. Mizwa (Kosciuszko Foundation, New York, 1945).

C. A. Lubbock, *The Herschel chronicle, the life-story of William Herschel and his sister, Caroline Herschel, edited by his granddaughter, Constance A. Lubbock* (Cambridge Univ. Press and Macmillan, 1933).

M. W. Burke-Gaffney, *Kepler and the Jesuits* (Bruce Publishing Co., Milwaukee, 1944).

R. Small, *An account of the astronomical discoveries of Kepler; including an historical review of the systems which had successively prevailed before his time* (J. Mawman, London, 1804).

E. Ford, *David Rittenhouse, astronomer—patriot, 1732–1796* (Oxford Univ. Press; Univ. of Pennsylvania Press, 1946).

Astronomers—Collected

A. F. Collins, *The greatest eye in the world; astronomical telescopes and their stories* (D. Appleton-Century, 1942). Contains much biographical material on the past and present directors of the principal observatories in the world.

E. J. C. Morton, *Heroes of science—astronomers* (Society for Promoting Christian Knowledge, London; E. and J. B. Young, New York, 1882). Copernicus, Brahe, Kepler, Galileo, Newton, Lagrange, Laplace and Herschel.

Alfred Noyes, *Watchers of the sky* (Stokes, New York, 1922). Copernicus, Brahe, Kepler, Galileo, Newton, W. Herschel and J. Herschel.

Selected Engineers and Chemists

J. H. Johnson and W. L. Randell, *Colonel R. E. B. Crompton* (Longmans, Green, London, 1945).

E. M. Brockbank, *John Dalton. Some unpublished letters of personal and scientific interest, with additional information about his colour-vision and atomic theories* (Manchester Univ. Press, 1944).

H. Lonsdale, *John Dalton, F.R.S.* (Routledge and Sons, London, 1874). This book is volume 5 of a set generally titled *The Worthies of Cumberland.*

F. T. Cooper, *Thomas A. Edison* (Stokes, 1914).

H. G. Garbedian, *Thomas Alva Edison: builder of civilization* (Julian Messner, 1947).

E. C. Kenyon, *Thomas Alva Edison, benefactor of mankind; the romantic life story of the world's greatest inventor* (John C. Winston, Philadelphia, 1931). Reissued in 1940 as *Thomas A. Edison; an inspiring story for boys.*

J. W. Hammond, *A magician of science; The boy's life of Steinmetz* (Century, 1926; D. Appleton-Century, ed. 2, 1943).

H. P. Maxim, *A genius in the family: Sir Hiram Stevens Maxim through a small son's eyes* (Harper, 1936).

R. H. Parsons, *The steam turbine and other inventions of Sir Charles Parsons* (Longmans, Green, 1943).

R. L. Douglas, *Leonardo da Vinci; his life and his pictures* (Univ. of Chicago Press, 1944).

J. W. Lieb, *Leonardo da Vinci, natural philosopher and engineer* (privately printed, 1921).

F. Arago, *Life of James Watt with memoir on machinery considered in relation to the prosperity of the working classes. To which are subjoined an historical account of the discovery of the composition of water by Lord Brougham, and an eulogium of James Watt by Lord Jeffrey* (A. and C. Black, Edinburgh, 1839).

H. W. Dickinson and H. P. Vowles, *James Watt and the industrial revolution* (Longmans, Green, London, 1943).

A. L. Mellanby, *James Watt, mathematical instrument maker to the University of Glasgow* (Jackson, Son and Co., Glasgow, 1936).

A. M. Pagan, *James Watt and the pioneer inventors* (Blackie and Son, London, 1928).

The story of Watt and Stephenson (W. and R. Chambers, London, 1892).

Proceedings of the public meetings held at Freemason's Hall on the 18th-June, 1824 for erecting a monument to the late James Watt (John Murray, London, 1824).

Life of James Watt, from Chamber's "Miscellany," 1871, contributed by D. J. Keir (Newcomen Society, New York, 1938).

The person who won't take advice isn't necessarily any more stubborn than the one who is offering it.—ANON.

Reproductions of Prints, Drawings and Paintings of Interest in the History of Physics

37. Rubens as a Scientific Illustrator

E. C. Watson
California Institute of Technology, Pasadena 4, California

WHILE Peter Paul Rubens (1577–1640), the great Flemish painter, exerted an enormous influence upon the art of engraving[1] and designed frontispieces for approximately 50 books, only seven or eight books are known with certainty to have been fully illustrated by him.[2] Fortunately, one of these seven is of considerable scientific interest, and Rubens' illustrations make it one of the finest scientific books of its period.

The *Opticorum libri sex philosophis iuxta ac mathematicis utiles* by Francis Aguilon is a thick folio volume of more than 700 pages published at Antwerp in 1613. As the title indicates, it consists of six books on optics. Each book is introduced by a large allegorical vignette engraved by Th. Galle from drawings made by Rubens. The work derives its importance from the facts that it lays the foundations of horopterology, provides excellent treatments of binocular vision, projections, and so on, and introduces the term "stereographic projection."

Francois d' Aguillon (1567–1617), as his name is given in the *Biographie Universelle*, was a Belgian Jesuit, who as a young man was Professor of Philosophy at Douay and later was Professor of Theology and Rector of the Jesuit college at Antwerp. He was one of the first to introduce mathematical studies into Flanders, and wrote a treatise on *Projections of the Sphere* in addition to the *Six Books on Optics*.

The first of the *Six Books on Optics* deals with "The Organ, Object[3] and Nature of Vision." Plate 1 reproduces the vignette that introduces this book. It shows cherubs dissecting the eye of a cyclops under the scrutiny of a teacher.

The second book deals with "Optical Rays and the Horopter." Its vignette (Plate 2) shows the same figures using a variety of optical instruments that depend upon the linear propagation of light. It was this book that laid the foundations of horopterology.

The third book discusses the perception of such common "objects" as size, shape, position, orientation, distance, continuity, discontinuity, motion, rest, and so forth. It is charmingly summarized by the vignette reproduced here as Plate 3.

The fourth book is entitled "Visual Fallacies" and the vignette (Plate 4) illustrates one of the many optical illusions that are discussed in detail.

Plate 1.

Plate 2.

[1] See H. Hymans, *La gravure dans l' école de Rubens* (Brussels, 1879).

[2] M. Funck lists only seven in his *Le livre Belge à gravures* (Paris and Brussels, 1925), p. 238.

[3] The term "object" is here used with its original meaning of that which presents itself to the senses. Thus light and color are "objects" of vision.

PLATE 3.

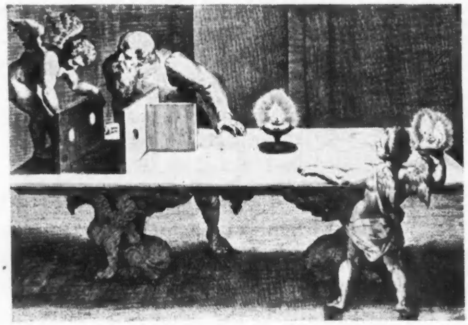

PLATE 5.

The fifth book deals with "Light and Shade." The vignette in this case (Plate 5) is of especial interest for it clearly portrays a photometer experiment. The invention of the photometer is usually credited to PIERRE BOUGUER (1698–1758), but here in an engraving executed more than a hundred years before BOUGUER's first publication we have a clear, as well as artistic,

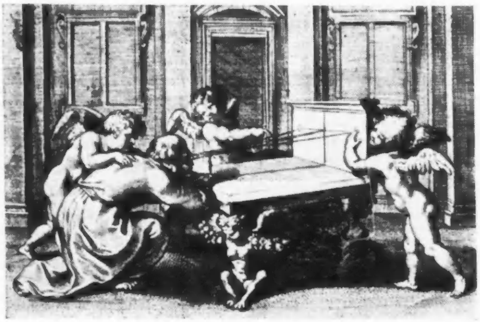

PLATE 4.

PLATE 6.

portrayal of a working photometer with all its essentials.

The sixth and last book occupies nearly a third of the whole volume. It is entitled "Projections" and deals with perspective and scenography as well as with both orthographic and stereographic projections. Its vignette (Plate 6), like the other five, is not only artistic but also quite appropriate to the text.

Taken together, these six beautiful engravings display a very considerable comprehension of the principles of optics on the part of one of the world's greatest and most prolific artists. The frontispiece to the whole work, which is also ascribed to RUBENS by most authorities, is, however, a fantastic composition in such poor taste and so full of absurdities that it does not warrant reproduction.

Young authors . . . always overestimate the capacity of their audience to grasp at short notice and in quick time ideas which they themselves have slowly and painfully evolved.—J. J. THOMSON.

NOTES AND DISCUSSION

Meaning of the Ratio e/m

P. LeCorbeiller
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THE following comment may serve as a footnote to "Jubilee of the electron."[1] Most textbooks give for e/m the value 1.759×10^7 cgsm. I never read these (or similar) results without wondering what meaning they can possibly have for the student. The objection is not that we are dividing two heterogeneous quantities; we are quite justified in dividing a mass by a volume, or a voltage by a current, because we have direct conception of unit density and of unit resistance. But who has a direct concept of the ratio of charge to mass?

Personally, I like at this point to tell my students the following. The Coulomb force between two electrons at a distance d(cm) is, in cgsm units,

$$F_{el}=c^2e^2/d^2,$$

and the gravitational force between them is

$$F_{gr}=Gm^2/d^2.$$

Hence,

$$\frac{F_{el}}{F_{gr}}=\frac{c^2}{G}\left(\frac{e}{m}\right)^2=\frac{8.99\times10^{20}}{6.67\times10^{-8}}(1.76\times10^7)^2=4.17\times10^{42},$$

a pure numeric. We have thus derived a result that is independent of the system of units used: the electrostatic force between two electrons is 4.17×10^{42} larger than the gravitational force. This result holds true, whether one favors Eddington's interpretation of it or not.

[1] E. C. Watson, *Am. J. Physics* **15**, 548 (1947).

Circuital Form of Ampère's Law: an Example

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THE general proof of Ampère's circuital law is based on the concept of the magnetic shell.[1] This general law can be nicely illustrated in a special case with which the student will already have considerable familiarity. This case is based on the law of Biot and Savart for the magnetic field of an infinitely long straight wire. In the simplest and well-known form, the magnetomotive force $4\pi i$ is at once obtained by integrating along a circular path concentric with and normal to the wire. This example can be generalized to include integration along any conceivable curve surrounding the wire.

First consider any plane curve whose plane is normal to the wire. Take the intersection of wire and plane as origin of polar coordinates r and θ of the curve. Let ϕ be the angle between $\mathbf{H}$ and the element $\mathbf{ds}$ of the curve at any point (r,θ). The magnetomotive force $\oint H\cos\phi ds$ can then be written

$$\oint\left(\frac{2i}{r}\right)\left(\frac{rd\theta}{ds}\right)ds=4\pi i.$$

It should be noted at this point that this result is valid for *any* plane curve, since $\oint d\theta$ must be equal to 2π for any closed curve.

The extension to any nonplanar curve follows from the fact that no work is done on the unit pole for components of displacement parallel to the wire. This means that the work done will be equal to the work done in moving over a plane curve which is the projection of the nonplanar curve on a plane normal to the wire, and hence equal to $4\pi i$.

[1] See, for example, Page and Adams, *Principles of electricity* (Van Nostrand, 1931), p. 258.

Concerning the Action of the Crookes Radiometer

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WHEN both vanes of a Crookes radiometer are exposed to luminous radiation the *light mill* rotates, the vane with the black side moving away from the light source. The explanation practically always given is, "the dark side is repelled more than the bright side."[1] In further explanation we picture the black side heated more than the bright side, the gas molecules near the black side, given a livelier motion, bounce away from the surface, and the back action causes that side to be "repelled more than the bright side" because the latter is not heated as much as the former.

It would follow from this reasoning that if light falls only on the bright side, that vane would be repelled from the source but not as strongly as would the dark side were it exposed. However, in the case of the radiometer in the physics building at Dartmouth College, when the bright side only is exposed the vane does not move away from the light source; its *motion* is *towards* the *source*. There is suction not pressure. When the black surface only is exposed, it moves away from the source. There is pressure not suction. *When both vanes are exposed, the motion is most rapid*. The explanation is this:[2] the vanes are not quite vertical. If the top of the vane whose bright side is towards the source is away from the source, the warmed, rising air current causes the vane to approach the source. When the light mill has turned through 180° about the vertical axis, the top of that vane is now *towards* the source and the convection air current causes repulsion, the action

being stronger because the black surface is now exposed to the light.

Whether pressure or suction takes place depends on (i) the pressure of the gas, (ii) the inclination of the vane, and (iii) the geometry of the whole system, including the walls of the vessel.

It would interest the writer to learn if others have found that the bright surface, when it alone is exposed to the light, approaches the source. Obviously for all such radiometers the ordinary explanation of radiometric action does not hold.

Note added in proof: There is another possible explanation; if the black surface is of such a nature that gas particles are absorbed, then a slight rise of temperature would cause particles to be ejected with a resulting back action.

[1] See J. Larmor, article on "Radiometer," *Encyclopædia Britannica* (ed. 11); Starling, *Electricity and magnetism* (1924), p. 421; Perkins, *College physics*, p. 260.
[2] G. F. Hull, *Physical Rev.* 20, 188, 292 (1905).

Electric Field-Mapping Apparatus

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"It is one of the lessons of the history of science that each age steps on the shoulders of the ages which have gone before."[1] A similar statement is most appropriate in discussing the simple method for mapping electric fields described herein, since this apparatus also evolved from earlier forms. Some years ago a "dry type" of field-mapping apparatus was introduced,[2] in which a sheet of material of high resistivity replaced the older electrolyte tray. This is an obvious improvement for equipment that must be handled by large groups of inexperienced students. Since that time other poorly conducting sheets have become available, including a "resistance paper" (Cenco) which is used in the apparatus described herein. The paper has quite uniform electrical characteristics; its minor variations may even enhance its instructional value. A single sheet can be used repeatedly.

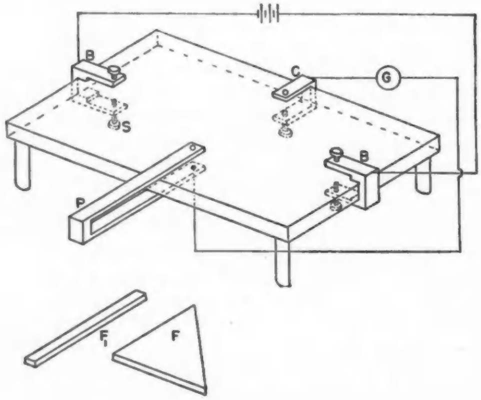

Fig. 1. Electrostatic field-mapping apparatus.

Figure 1 shows the plan of the apparatus. It consists of a 9×12-in. presswood panel mounted on 1½-in. legs. The resistance paper is attached to the underside of this board. Scotch tape may be applied around the edge of the paper to hold it in place. The data record sheet, cross-ruled paper if desired, rests on the top surface of the panel.

The field voltage is applied to the fixed terminals B, which make contact with the resistance paper through the screws S. Sufficient clearance is provided to clamp the "field units" F to the paper. By using two conducting bars (F_1), a parallel-plate field may be studied. Similarly, many other models may be used. Since the top screws are located directly above the bottom ones, the model may be readily first traced on the record paper. One may even bend models from bare copper wire and attach them directly to the resistance paper with Scotch tape. It was suggested to the author that the resistance paper might be cut in a pattern to facilitate the study of flow problems.

The balancing-potential clamp C makes contact by a screw with the resistance paper. A hole in the upper arm directly above this screw locates the corresponding position on the record paper. The U-shaped[3] exploring probe P, which is made of Bakelite, has sufficient spring to hold the metallic contact point against the resistance paper as the probe is moved about. When the null point is found, the student locates this position on the record paper by circling the hole cut in the upper arm, which is directly above the contact position. Thus no transfer of data is necessary, and the student can, in the usual laboratory period, make studies of several field arrangements.

A field potential of 6 v and a Weston 440 galvanometer on the exploring probe will enable one to locate equipotential points to within 1 mm. Other means of detecting equipotential points may, of course, be used. This one is suggested for its simplicity and the availability of the additional apparatus.

[1] Sir Michael Foster.
[2] Central Scientific Company, Electric fields of force apparatus.
[3] A. D. Ehrenfried, *Am. J. Physics* 12, 371 (1944).

To Determine the Greatest Depth in Water at which One Can Breathe through a Tube

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In any physics class there are enough students interested in swimming and related topics to insure interest in deep-sea diving and underwater swimming. One question of interest concerns the reason for the use of complicated diving suits. Why not just breathe through a sturdy hose whose upper end is attached to a float? The fact that the water pressure on one's chest will prevent inhalation, even at shallow depths, seems a bit hard to believe, even when numerical values are presented as proof. It is desirable to let a person determine this critical depth by an experi-

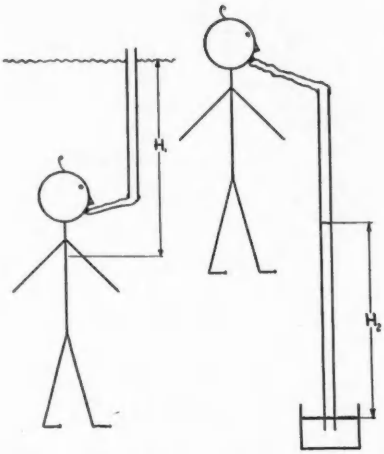

FIG. 1. Method of testing the strength of the chest muscles.

ment, but obviously it is inconvenient to try the direct method.

An equivalent and rather striking experiment is to test the strength of the chest muscles by measuring the height to which a column of water can be drawn up through a long glass tube by inhaling. One must use a normal air intake action and avoid sucking the liquid up by using the tongue and cheek muscles, as the latter method is not the one by which air is inspired, and it gives an erroneous impression by producing an unusually low pressure in the mouth. The vertical height of the column is exactly the depth to which our diver could go and still breathe through a tube. The experiment in part owes its effectiveness to the direct appearance of the result without recourse to any calculations. This depth would, of course, be measured to some undetermined average position on the person's chest, but the result is still rather startling even with this small uncertainty present. The possible enormity of hydrostatic forces seems to become somewhat clearer.

We can see that the two distances H_1 and H_2 in Fig. 1 are equivalent by the following argument. Depth H_1 measures the increase in pressure on the chest and abdomen above atmospheric pressure. Height H_2 measures the decrease, below atmospheric, in the pressure exerted on the inside of the chest. Since it is the difference in force, and thus pressure, on the inside and the outside of the body that hinders taking in a breath, H_1 and H_2 should be the same. This argument can be elaborated upon or made more exact, but the essentials have been indicated.

After having observed a number of people perform this experiment, it seems safe to state that few persons will be able to raise a column of water even 5 ft. This means that one's head could only be about 3 ft under the surface of the water. We should note that these values are for a single very strenuous and tiring inspiration. In the case with the water in the tube, there seems to be no limitation imposed (or danger introduced) by a tendency for the lung sac to come loose from the chest wall. Tests performed in a swimming pool seem to verify these conclusions. For example, one outstanding subject who reached slightly beyond the 5-ft mark, found it extremely difficult to breathe when his head was 3 ft below the surface of a fresh water pool. When lowered another foot he found it impossible.

This case, when its significance is clear, should be contrasted with the more usual diving outfits, with native divers swimming freely and without danger of collapse at 30-ft depths, and with the possibility of breathing from a submerged balloon or from under a fishbowl inverted over the head.

Surface Energy and Surface Tension

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ELEMENTARY textbooks of physics, when they include any discussion of surface tension at all, are likely to include a picture in which a molecule deep in the liquid is shown as a dot with arrows pointing out symmetrically along radii of a circle drawn around it to represent forces of attraction by neighboring molecules, and in which a molecule in the surface is also shown, supplied with only half a circle beset with arrows. It may even be stated in so many words that there is a resultant force on the surface molecule directed into the liquid.[1]

A little reflection shows that surface molecules, like interior molecules, must, on the average, be in static equilibrium under forces applied to them by their neighbors, and an unwary student (or teacher) is likely to conclude that the unsymmetrical set of attractions shown in the picture has to be balanced by a special—and unpictured—repulsive force, normal to the surface, to be associated in some mysterious way with a set of tensions in the surface—also not pictured. This leads in turn to the idea, wholly false, that the surface layer of a liquid must be compressed normally and extended laterally, like an elastic solid membrane. Actually, the mean separation of such molecules as can be represented as spheres in a simple theory must increase as a free surface is closely approached, no matter what direction between neighbors is considered. At the final transition from liquid to saturated vapor the mean separation must further increase to a value consistent with the vapor density at the temperature considered.

One way of presenting a better set of ideas is to point out that well inside a liquid, under zero external pressure, each molecule is, on the average, in equilibrium (in a potential-energy valley) where forces of attraction to its neighbors and repulsion from its neighbors just balance. If pressure is now applied, the mean separation of molecules gets less, the repulsions between them increase and the liquid gains potential energy with respect to its configuration at zero pressure. If we apply the pressure by letting part of the liquid evaporate in a closed vessel which the liquid cannot fill at zero pressure, the interior of the liquid will acquire a definite potential energy per unit volume,

characteristic of the substance and the temperature. Near the surface of separation between liquid and vapor the liquid has a lower density and a different potential energy per unit volume, both of these depending upon distance from the surface. The volume density of potential energy at a point just below the surface must be higher than that at an interior point. Without knowing in detail how the potential energy varies with distance from the surface, we are, therefore, safe in assigning a positive value to the excess potential energy of all the surface layers per unit of surface area. In other words, it takes work to produce additional surface with a fixed volume of liquid.

Another way of reaching the same conclusion about the sign of surface potential energy is to notice that part of the work against molecular forces which has to be done in vaporizing a liquid has already been done in separating its molecules a little at a free surface.

Energy per unit area can be expressed in several ways: in ergs per square centimeter, in dyne·centimeters per square centimeter, in dynes per centimeter, and so forth. It is thus equally correct to specify the surface energy of a water-water vapor boundary as 75 erg/cm^2 or to specify it as a surface tension of 75 dyne/cm. It will not do, however, to specify the surface energy as 75 g/sec^2, though this has the same dimensions as the two preceding quantities. The reason for objecting to this last simplification is that the units [erg/cm^2] and [dyne/cm] still retain enough length factors to identify them as tensors, whereas the unit [g/sec^2] has lost its hold on a peculiar plane in space where area must lie in the first expression, and in which both force and length must lie (at right angles to each other) in the second expression.

The notion of surface energy is superior to that of surface tension when we consider such things as wetting of solids by liquids, mutual solubility of liquids, and conditions at a critical point. The notion of surface tension is superior when we consider pressure differences across curved interfaces, the shapes of bubbles, the statics and dynamics of liquids in capillary tubes and other matters where the simple plane surface is geometrically inadequate. By introducing both notions at the start, as two names for the same quantity, the choice in a new case is made much easier.

It is probably too much to hope that textbooks will ever be able to avoid all arguments which contradict each other when carried to logical conclusions. It would help, however, if unbalanced forces were assumed to exist only where accelerations can be found to accompany them.

[1] See, for example, R. L. Weber, M. W. White and K. V. Manning, *College technical physics* (McGraw-Hill, 1947), p. 218. (This text is especially free from faults of the kind here in question.)

The First Excitation and Ionization Potentials of Mercury

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THE procedures that have been suggested for the measurement of the excitation and ionization potentials of mercury as a laboratory experiment for advanced undergraduate students ordinarily demand a specially constructed tube.[1,2] Following a suggestion[1] that a commercial tube, the FG-67 thyratron, may be used, we have found that the circuit illustrated in Fig. 1 yields satisfactory results. Provision must be made for maintaining the tube at a temperature of approximately 100°C. It is sufficient to wrap a towel about the tube and use the heat developed in the cathode heater to maintain the desired temperature. A neater method is to place the tube in a small electrically heated oven. The plate lead of the tube should be well insulated. The plate current is of the order of 10^{-9} amp and may be measured with a simple triode amplifier. A battery operated amplifier using the Victoreen Vw-41 tube gives excellent results.

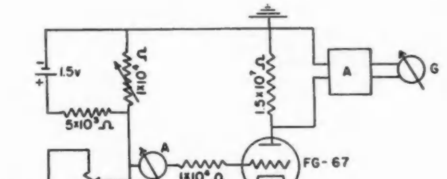

Fig. 1. Diagram of circuit.

A typical excitation curve is shown in Fig. 2. The difference between the grid potentials at which the first and second maximums in the plate current occur yields a

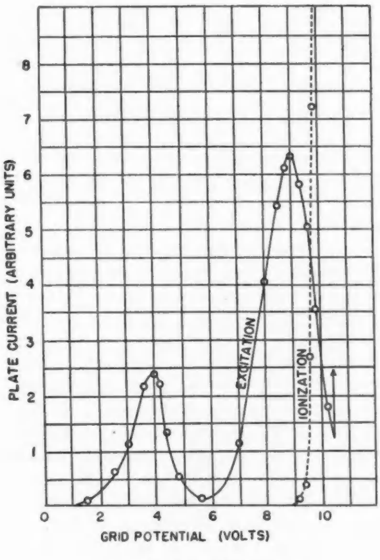

Fig. 2. Typical excitation curve.

value for the first excitation potential of mercury that is within 3 percent of the accepted value. The contact potential difference between cathode and grid may be determined by subtracting the first excitation potential from the grid potential at which the first maximum in the plate current occurs.

The ionization potential is determined by making the plate 60-v negative with respect to the grid and measuring the positive ion plate current as a function of the grid potential. The tube temperature should be reduced to approximately 50°C in order to increase the mean free path of the electrons. The grid-cathode contact potential must be subtracted from the grid potential at which ionization occurs in order to obtain the ionization potential. The ionization potential measured in this manner agrees with the accepted value to within a few percent.

The cost of the FG-67 tube is $22.00 and the chief disadvantage of the experimental procedure here outlined.

[1] R. Hofstadter, *Am. J. Physics* **10**, 112 (1942).
[2] D. Fahey and J. G. Winans, *Am. J. Physics* **11**, 289 (1943).

"Equations for Straight Lines"—A Reply

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In his note on "Equations for straight lines," Paul F. Gaeher[1] makes some statements which seem to me to be very misleading. He discusses a common substitute for the method of least squares for determining the slope of a straight line when "one set of coordinates is free from experimental error and forms an arithmetic progression." The method is essentially as follows.

An even number ($2m$) of equally spaced ordinates whose true values are theoretically separated by a constant difference Δ are observed. If l_i denotes the observed value of the ordinate whose abscissa is $i[=1, 2, \cdots, 2m]$, and if y' is the adopted value of Δ,

$$my'=(\sum_1^m l_{m+i}-\sum_1^m l_i)/m.$$

The author then states essentially that (1) accidental errors in each of the groups $\Sigma\, l_{m+i}$ and $\Sigma\, l_i$ are *apt* to cancel out among themselves, (2) systematic errors in the one group cancel against those in the other, and, finally, (3) "Under the conditions specified this method is *rigorously correct*." (Italics mine.) While this method is a very reliable and useful short cut, I shall endeavor to prove that statements (1) and (3) are contrary to fact if the author means by *rigorously correct* giving the most probable value to be obtained from the data.

In considering the relative reliability of the two methods we are concerned only with how the observational errors affect the derived quantities. How *accidental* errors are apt to be propagated in the long run is susceptible of exact mathematical computation which does not involve the assumption of any particular law of distribution of errors. The only necessary assumptions are that (1) each error is only a small fraction of the corresponding observed quantity, and (2) positive and negative errors are equally likely to occur.[2] Since only linear functions are involved in the present problem, the first assumption is unnecessary, and the second is one of the basic assumptions as to the nature of *accidental* errors. The numerator of the expression for y' is an algebraic sum and, as is well known, its mean square error is the sum of the mean square errors of the individual terms. As these are all essentially positive they are not *apt to cancel among themselves*. How a systematic error in the one group would affect that in the other seems to me to depend entirely on the nature of the systematic error involved.

If x and y are, respectively, the "least squares" values of the first measured ordinate and of Δ, any residual is given by

$$x+(i-1)y-l_i.$$

If use is made of the well-known formulas for the sum and sum of squares of the first n integers, the least squares solution for y is

$$y=\frac{6\sum_1^{2m}(i-1)l_i-3(2m-1)\sum_1^{2m}l_i}{m(4m^2-1)}.$$

The first summation is the absolute term in the normal equation for y, and the weight of y, w_y is the reciprocal of the coefficient of this summation, so that

$$w_y=m(4m^2-1)/6.$$

The quantity y' is a linear function of the l's, and hence its weight is

$$w_{y'}=\tfrac{1}{2}m^3$$

and

$$w_y/w_{y'}=\tfrac{1}{3}\left(4-\frac{1}{m^2}\right),$$

which converges to 4/3 as the number of observations is increased indefinitely. For the minimum of two observations ($m=1$), both methods necessarily agree, but for a large number of observations the odds are *slightly* in favor of the method of least squares. Furthermore, the shorter method is only applicable to an even number of observations. If an odd number be made, the first or last of the series must be thrown out, which would be hard to justify.

I heartily agree with the author that the method he advocates is shorter and easier than the method of least squares, and I think I have demonstrated that it is nearly as reliable (odds better than three to four) for a limited number of observations. But it seems to me misleading to call it *rigorously correct*. Nor can I agree with the author's assumptions as to the canceling of the errors in the shorter method.

While claiming no originality for this treatment, I have never happened to run across an analysis of the relative merits of the two methods.

[1] P. F. Gaehr, *Am. J. Physics* **15**, 430 (1947).
[2] R. T. Birge, *Am. J. Physics* (*Am. Physics T.*) **7**, 351 (1939).

New Members of the Association

The following persons have been made *members* or *junior members* (*J*) of the American Association of Physics Teachers since the publication of the preceding list [*Am. J. Physics* 16, 120 (1948)].

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